Moonshot Goal 8
Realization of a society safe from the threat of extreme winds and rains by controlling and modifying the weather by 2050.

Actuator Location Optimization for Large Degree-of-Freedom Fields

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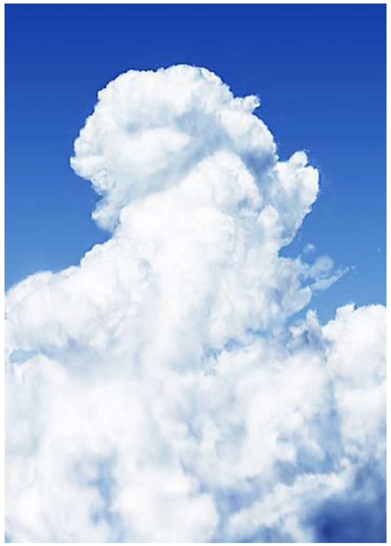
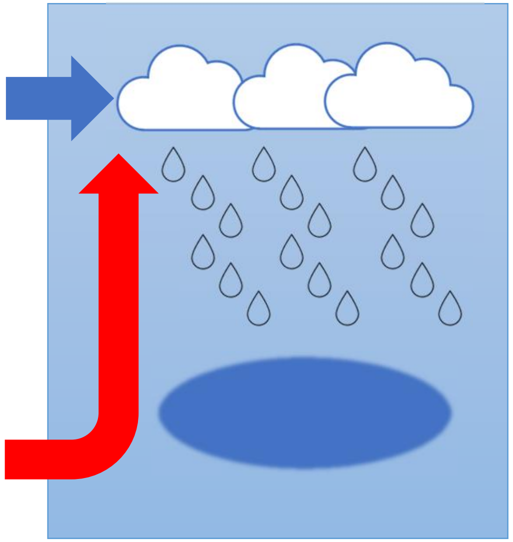
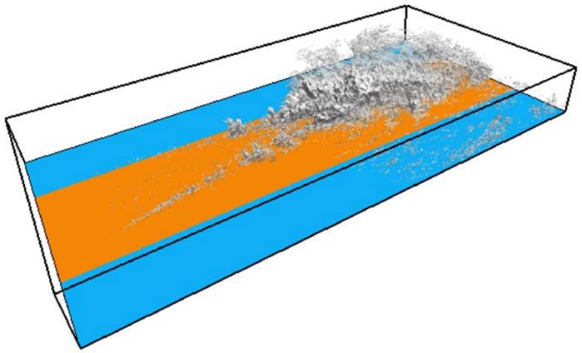
Summary of R&D projects (bottlenecks)

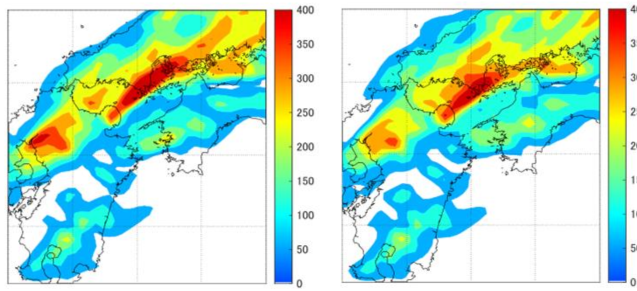
- Control methods and difficulties
  - Manipulating heat
  - Manipulating momentum
  - Manipulating phase transitions
    - Need to significantly improve the effectiveness of these weather control interventions



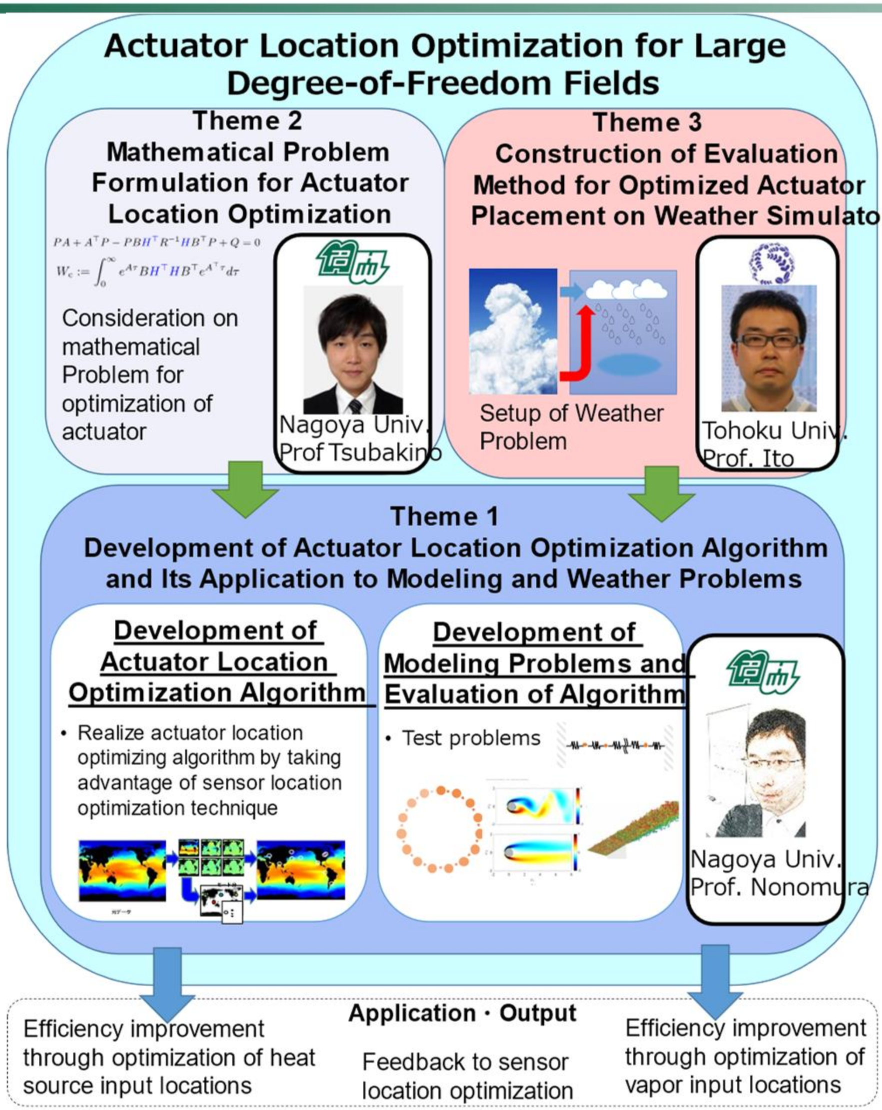


- Need to organize, evaluate, and develop optimization methods for actuator locations to significantly improve weather control effectiveness
  - Indices of where heat sources and water vapor should be placed
  - Appropriate control input points significantly improve input effectiveness and lead to successful weather control





Research themes and public relations activities

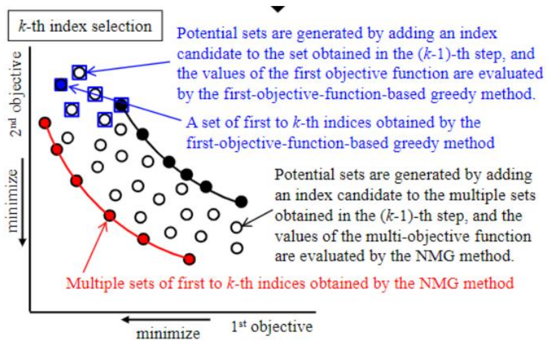


- **Website**

- **Symposium and conference activities**
  - Joint OS with Onishi G at IFAC2023
  - OS at Joint Conference on Automatic Control
  - Upgrading the research group on fluid control of the Society of Instrument and Control Engineers to launch the research group on fluid and weather control

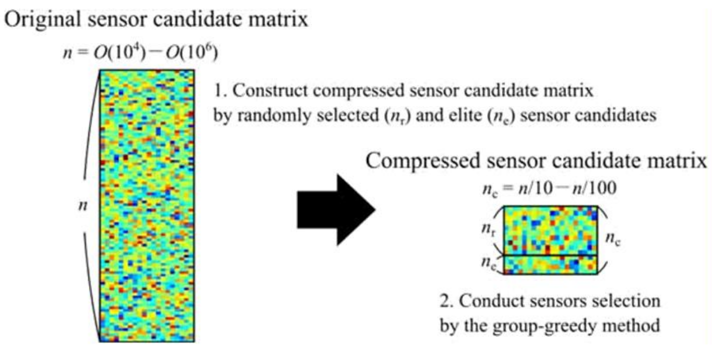
Sensor Optimization (can be used for actuator location optimization as a dual problem)

◆ Multi-objective greedy method



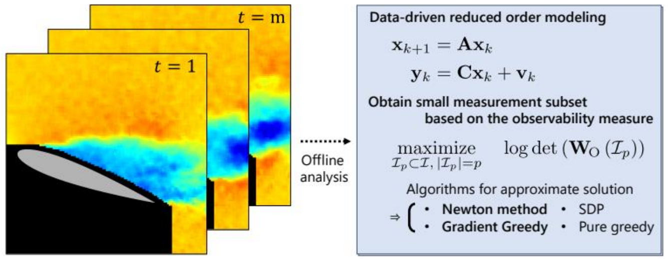
Nakai et al. *IEEE TSP*, 2023
Saito et al. *IEEE Sens. J.* 2024

◆ Randomized greedy method



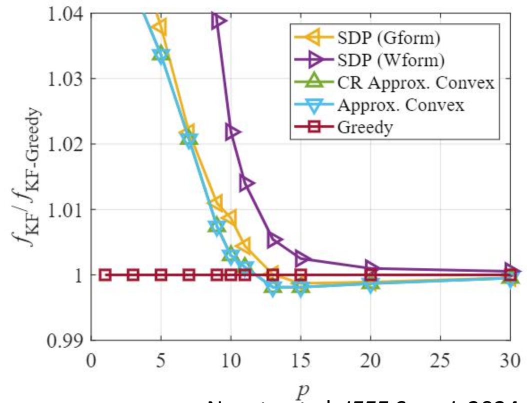
Nagata et al. *IEEE Sens. J.* 2023

◆ Sensor optimization based on observability Gramian



Yamada et al. *Sensors*, 2023

◆ Sensor optimization based on error covariance of Kalman filter



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◆ Comparison of indices for sensor optimization in dynamic systems

- ➢ Fisher information matrix
- ➢ Observability Gramian
- ➢ Kalman filter error covariance matrix

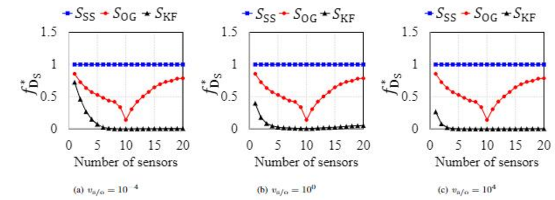

FIGURE 1. Normalized objective functions with respect to the selected number of sensors in a random system.

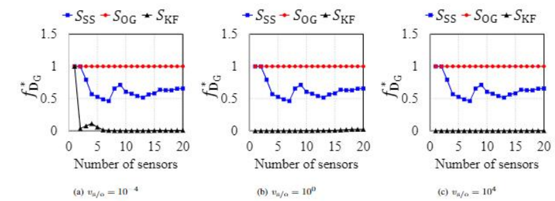

FIGURE 2. Normalized functions for the selected number of sensors in a random system.

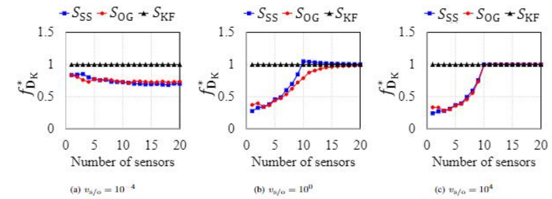

FIGURE 3. Normalized functions for the selected number of sensors in a random system.

Takahashi et al. *IEEE Access* 2023

Mathematical Obj. Function of Actuator Optimization

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- Relation of placement problems for linear time-invariant systems

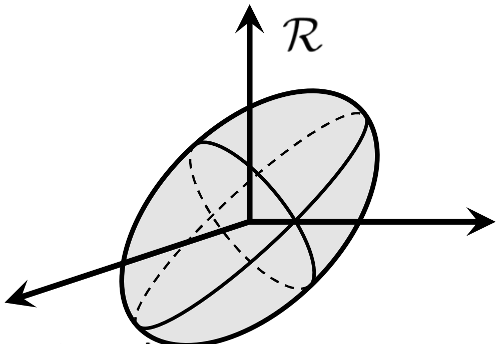
$y_n = Cx_n + w_n$ Linear inverse problem	$x_{n+1} = Ax_n$ $y_n = Cx_n + w_n$ Observability Gramian	$x_{n+1} = Ax_n + v_n$ $y_n = Cx_n + w_n$ Kalman filter
1-step ahead reaction $x_n = Bu_n$	Reachability Gramian $x_{n+1} = Ax_n + Bu_n$	LQR $x_{n+1} = Ax_n + Bu_n$ $z_n = [x_n^\top \quad u_n^\top]^\top$

- Objective functions of actuator location optimization problems
  - Tangent-linear-model-based approximation

TLM: $x[k+1] = A[k]x[k] + B[k]u[k], \quad x[0] = 0$

Objective: Maximize reachable set $\mathcal{R} = \{\, \xi \in \mathbb{R}^n \mid \xi^\top W^{-1} \xi \leq 1 \,\}$

Low dimensionality using singular vectors enables implementation in large systems



  - Type and magnitude relation of indices based on the evaluation method of matrix volumes

$$\mathrm{Tr}(W) \geq \lambda_{\max}(W) \geq \frac{\mathrm{Tr}(W)}{n} \geq \sqrt[n]{\det(W)} \geq \frac{n}{\mathrm{Tr}\left(W^{-1}\right)} \geq \lambda_{\min}(W)$$

Most robust

Actuator Optimization for Linearized Ginzburg-Landau Model

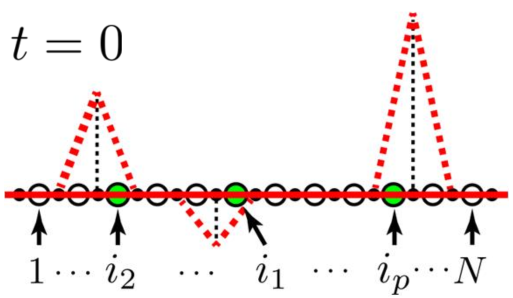
◆ Linearized Ginzburg-Landau model

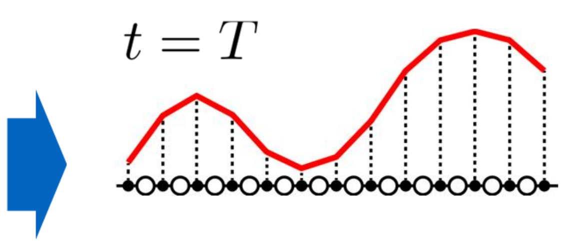
$$\frac{\partial q(x,t)}{\partial t} = \left(-\nu \frac{\partial}{\partial x} + \gamma \frac{\partial}{\partial x^2} + \mu(x)\right) q(x,t) + \underbrace{\sum_{i=1}^{p} \exp\left(-\frac{(x - x_{a_i})}{2\sigma^2}\right) \alpha_i \delta(t)}_{\text{input}}$$

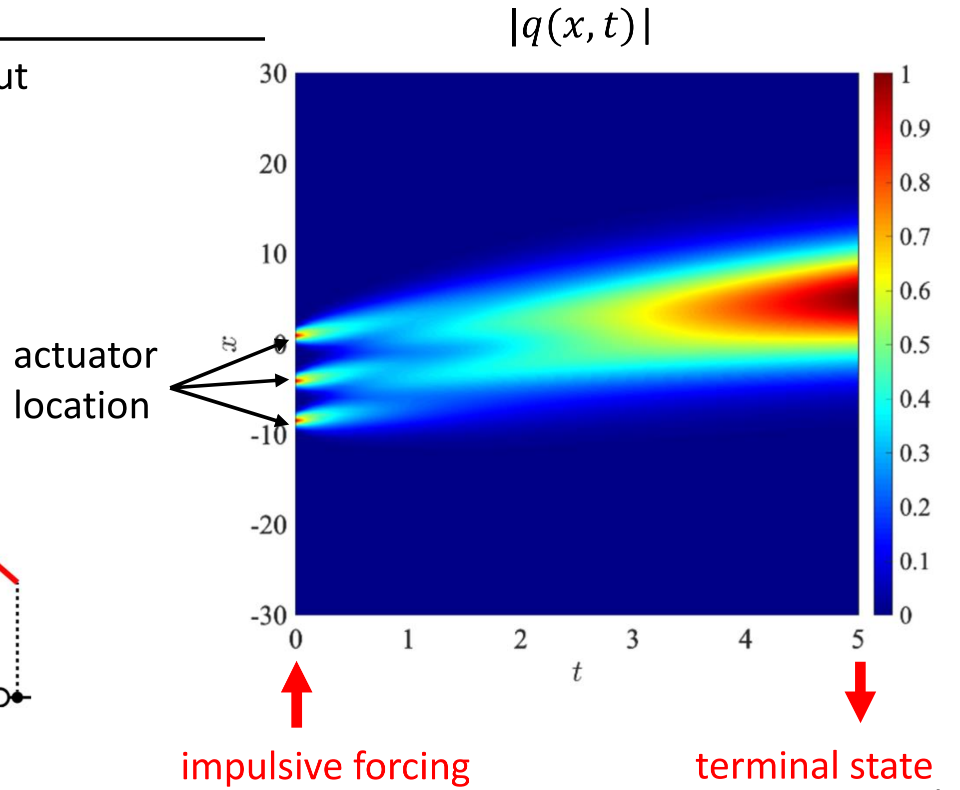
$$\frac{dy(t)}{\partial t} = (-\nu D_1 + \gamma D_2 + M) y(t) + Bu(t)$$

◆ Problem

Where should the actuators be placed so that $\|y(T)\|_2$ is maximized?



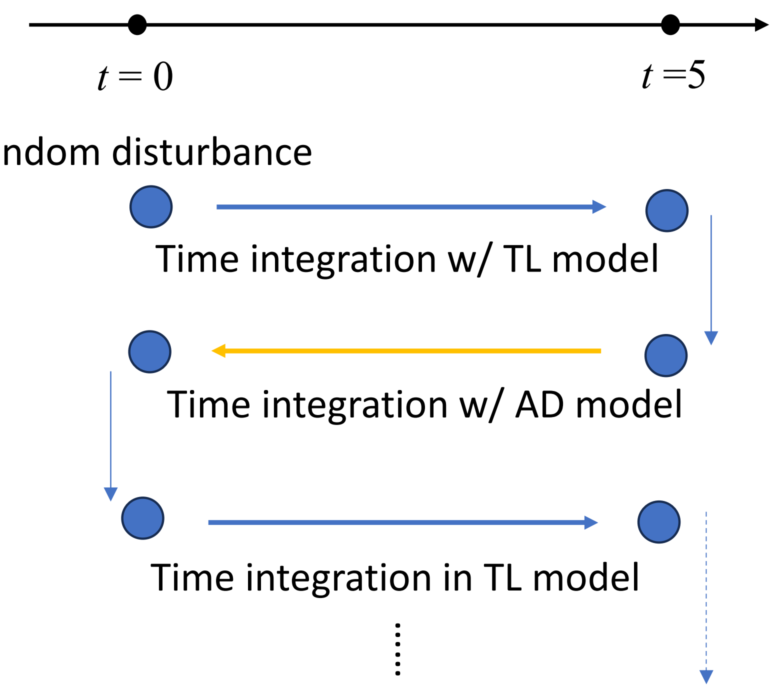




Optimization Method of Actuator Location

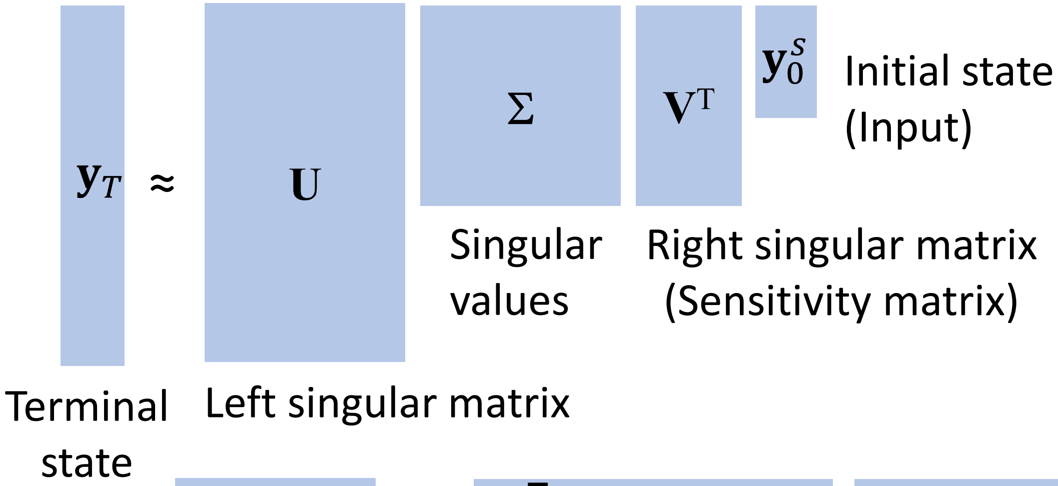
◆ Reduced order modelling by singular vector method

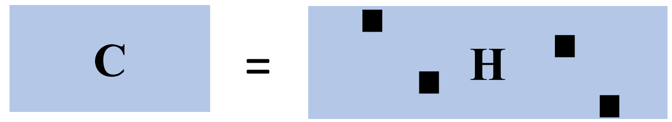
$y(T) = \Phi y(0)$ $\qquad$ $y^*(0) = \Phi^* y^*(T)$



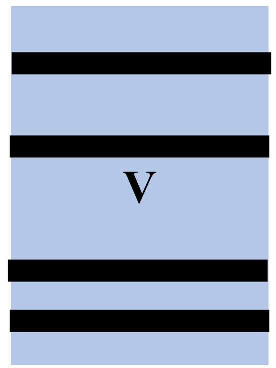
$\Phi \approx U_r \Sigma_r V_r^*$

◆ Actuator selection by greedy method





Actuator location matrix



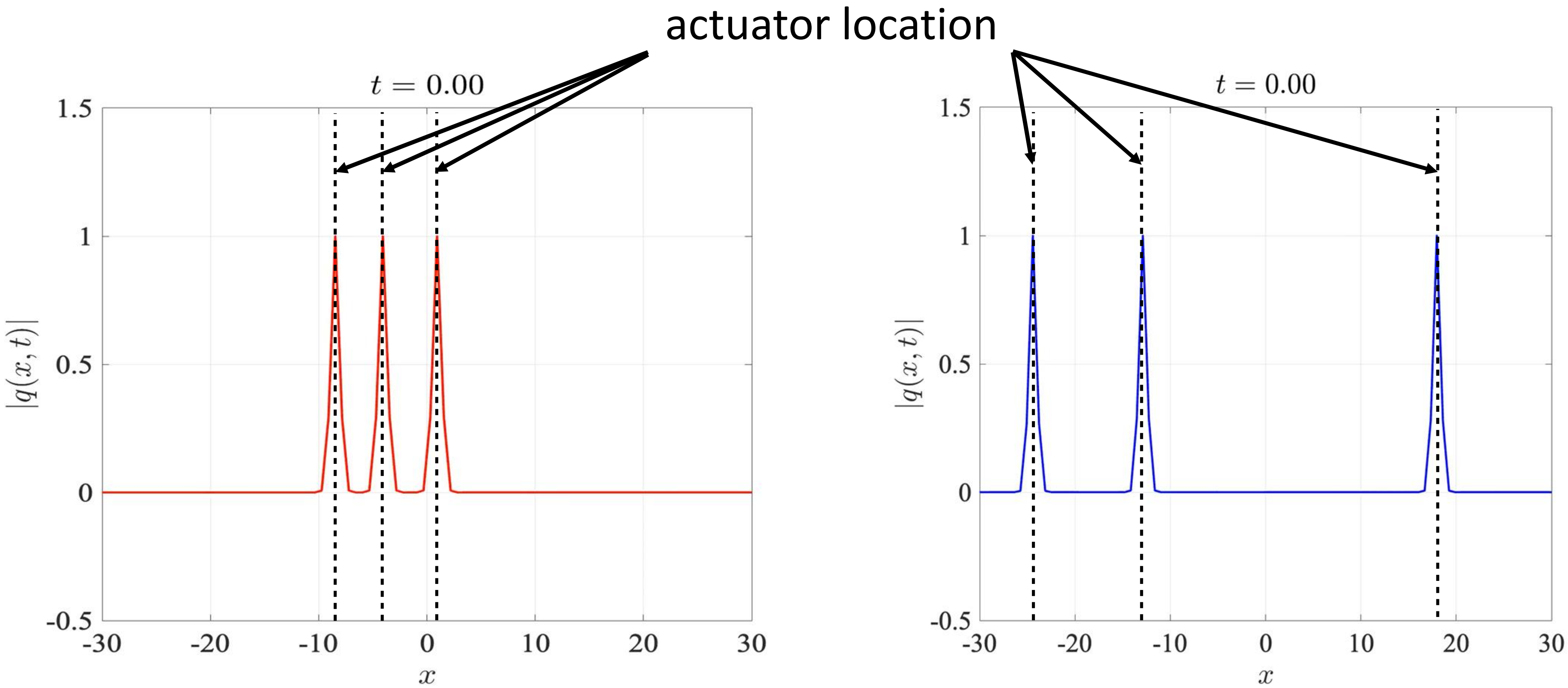
Right singular matrix

Let $\tilde{C} = C\Sigma$, then

$$\|y_T\|_2^2 = {y_0^s}^{\mathrm{T}} \tilde{C}\tilde{C}^{\mathrm{T}} y_0^s.$$

Thus, $\det(\tilde{C}\tilde{C}^{\mathrm{T}})$ is (sub)maximized.

Response by Impulsive Forcing



(Sub)optimized placement

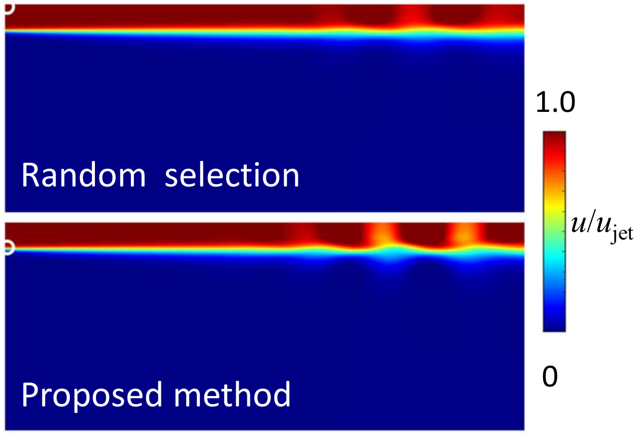
Random placement (One case)

Application to Fluid Problems and Lorenz Models and Prospects for Weather Control Problems

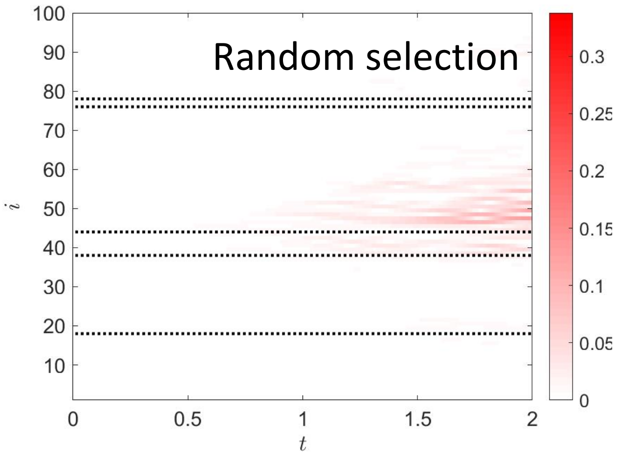
◆ Application to fluid (axial jet)

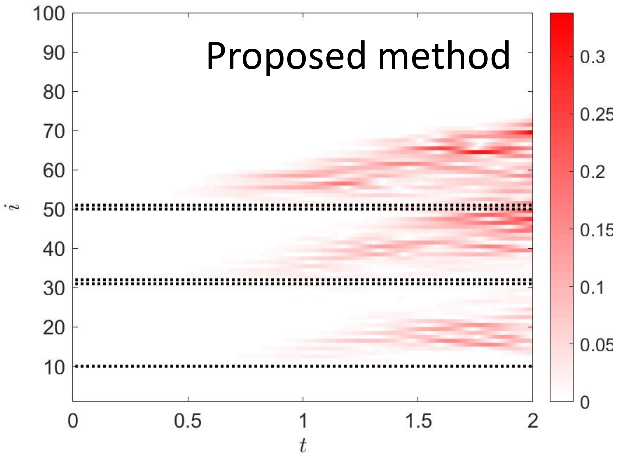
Left singular vector
(response mode)

Right singular vector
(Sensitivity mode)

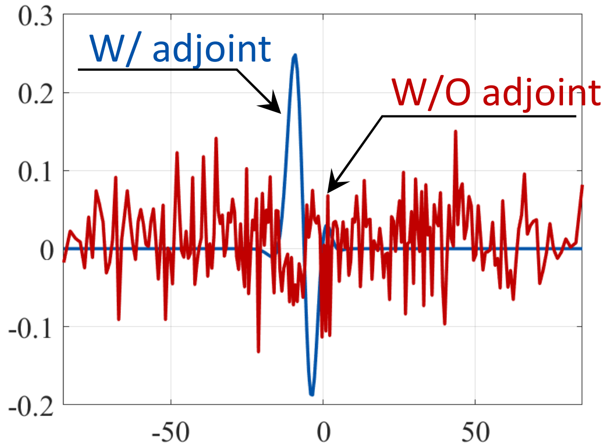


◆ Application to Lorenz 96



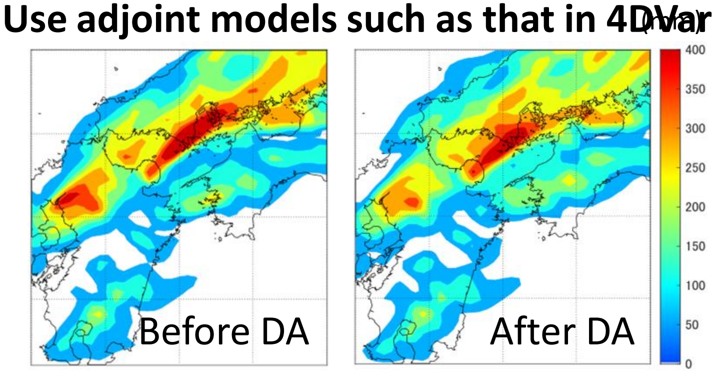


◆ Prospect to weather control Problem



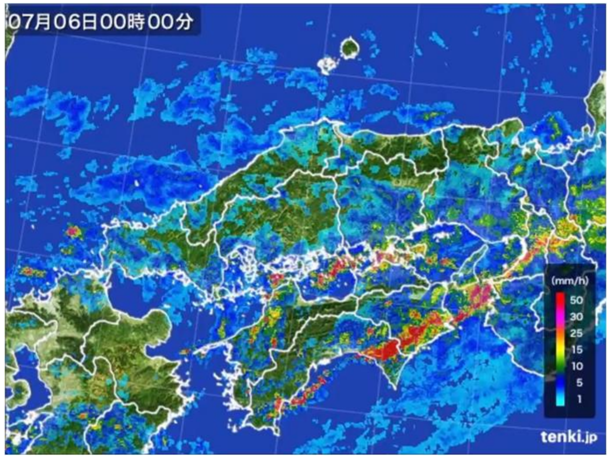
Difficult to calculate accurate sensitivity without an adjoint model

→Use adjoint models such as that in 4DVar



Application to Weather Modification

- Numerical simulation on the heavy rains in western Japan from June 28 to July 8, 2018
  - Analysis of the six-hour period from noon to 6 p.m. (UTC) on July 5, just before the peak rainfall on July 6.
  - Input: removing water vapor of at the actuator positions on the ground at the initial state
  - Objective: minimize the maximum accumulated precipitation



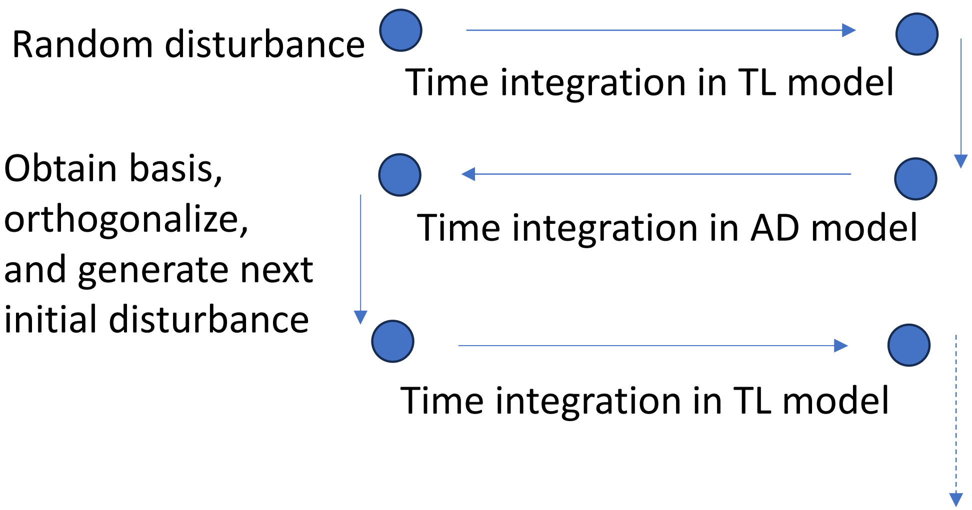
Objective function for control decision $$\min\left(\max\left(\mathbf{y}_T + \mathbf{y}_{\mathrm{base},T}\right)\right)$$

- Weather simulations
  - WRFPLUS V3.9.1.1
    - Developed by the U.S. Aeronomy Research Laboratory and others
    - Many physical processes available
    - Linear tangent and adjoint models available
  - Computational conditions
    - Horizontal grid resolution: 10 km
    - Number of grid points: 50 x 50 horizontal, 31 vertical
    - Time increments: 60 s

Implementation of singular vector method

◆ Lanczos method-based (standard)

- ➢ Tangent linear and adjoint analyses are repeated to generate matrices that approximate the properties of the system.

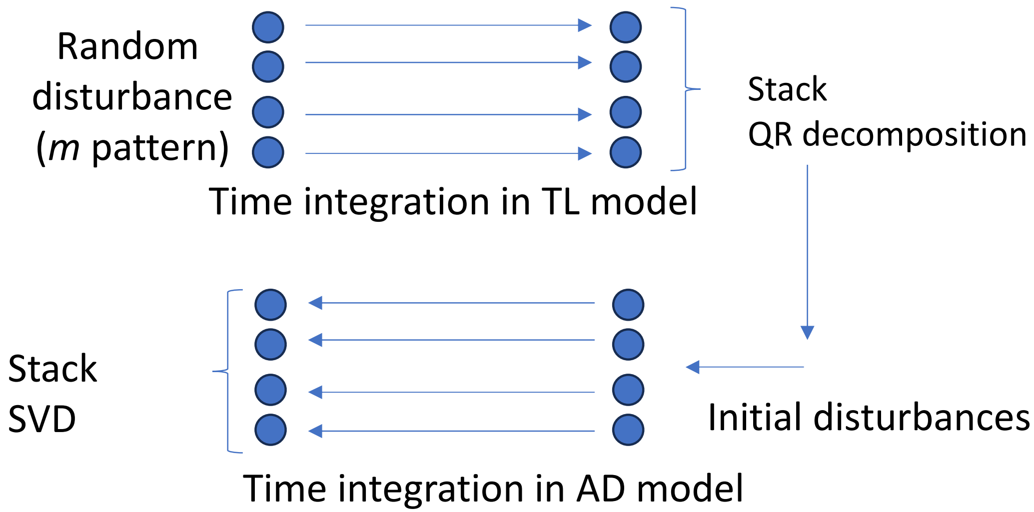


- ✓ Iterate over the number of singular vectors
- ✓ Computational load is large when the cost of time integration is large

◆ Randomized-SVD-based (proposed)

- ➢ Generates matrices that approximate the properties of the system with the results of multiple tangent linear and adjoint analyses
- ➢ Algorithm is consistent with randomized singular value decomposition

(Halko et al., 2011)



- ✓ Generates as many initial fields as the number of singular vectors required
- ✓ High parallel performance owing to independent execution of time integration

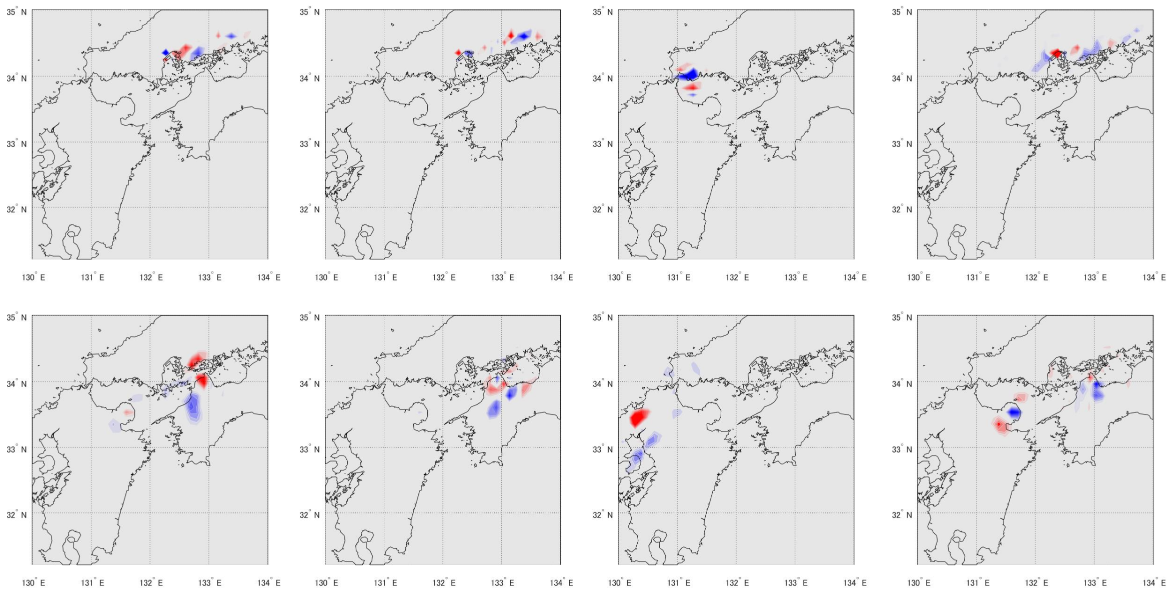
Obtained singular vector and actuator positions

Singular vectors obtained by the randomized singular vector method
(k = 32, m = 32)

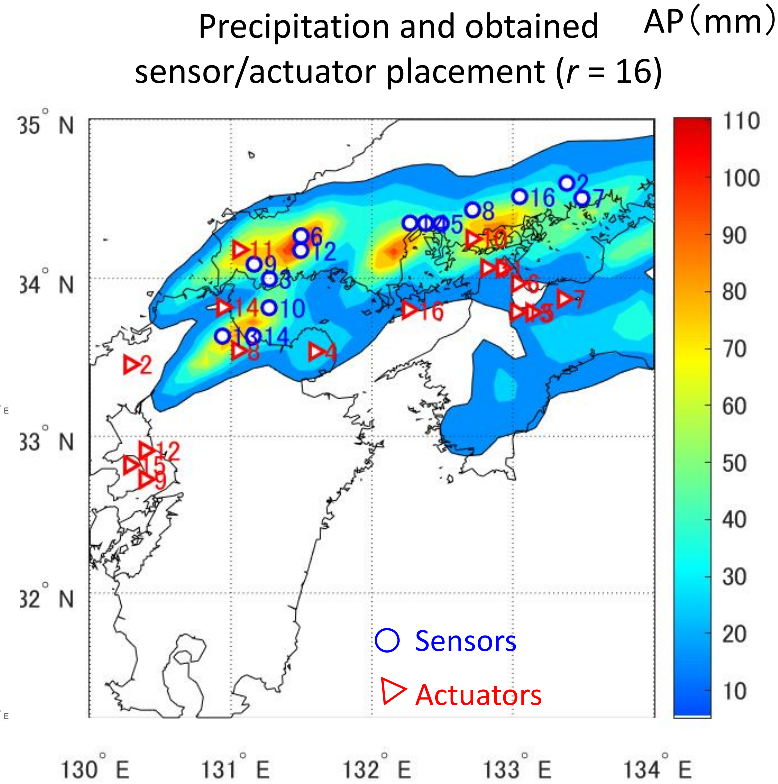
1st mode | 2nd mode | 3rd mode | 4th mode

Response mode (accumulated precipitation)

Sensitivity mode (WVMR on ground)



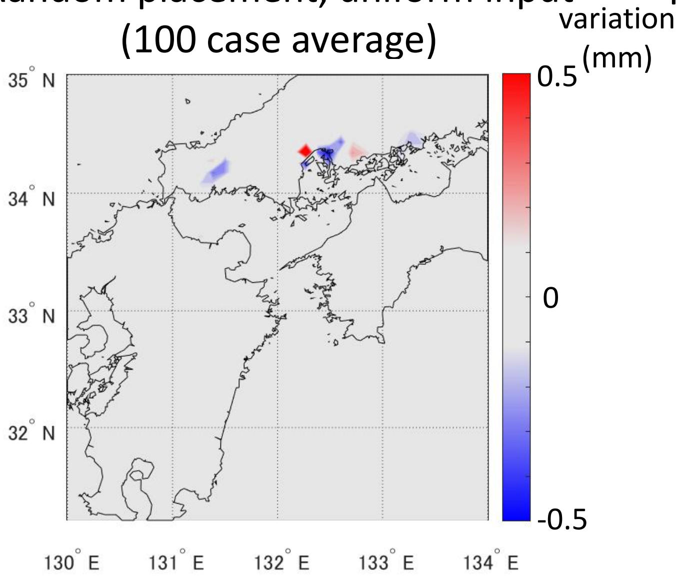
Precipitation and obtained sensor/actuator placement (r = 16)



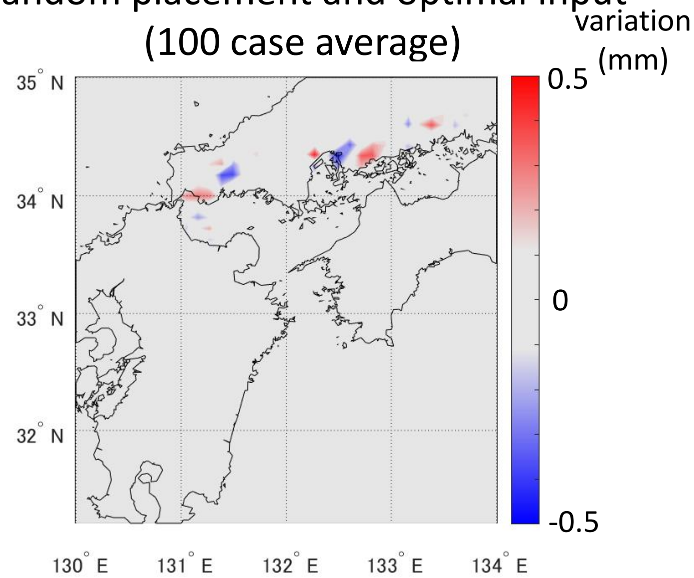
- ✓ No mode to simply increase/decrease rainfall
- ✓ Actuators were placed mainly on the southwest (upstream) side of the strong precipitation area

Control experiment with tangent linear model (*p*=16, reduction in maximum precipitation)

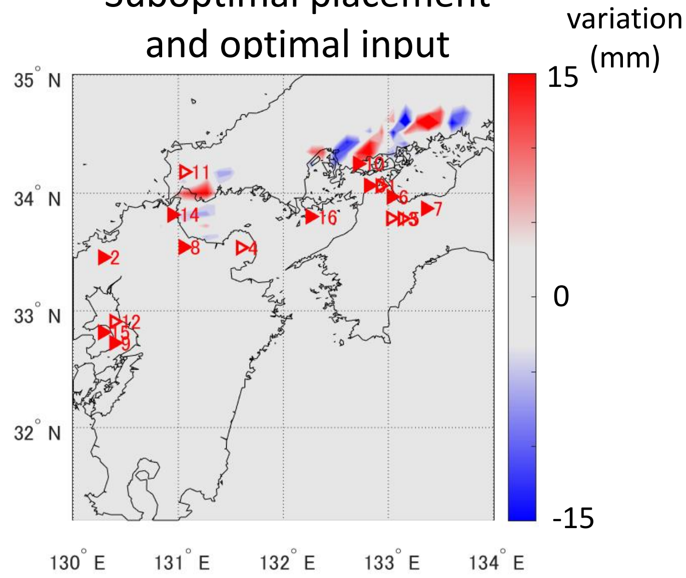
Random placement, uniform input (100 case average)



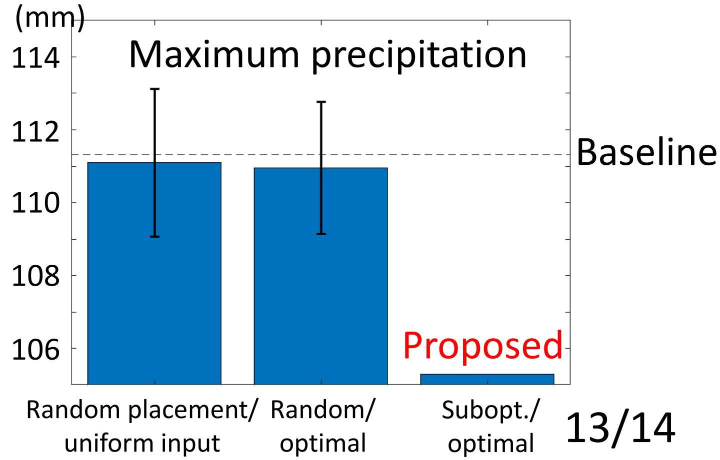
Random placement and optimal input (100 case average)



Proposed
Suboptimal placement and optimal input



- The suboptimal placement achieves the smallest maximum precipitation.
  - Optimization of the input has little effect on suppressing maximum precipitation for randomly placed actuators.
- Precipitation increased more than 10 mm/6h in some places.
  - It is necessary to carefully consider the objective function from the perspective of minimizing damage caused by rainfall.



Conclusions and Future Research

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An actuator location optimization framework for efficient control of large-scale degree-of-freedom fields such as weather fields is proposed and validated.

- ✓ The sensor location optimization methods were developed as dual problem of actuator location optimization.
- ✓ The mathematical interpretation was given and objective functions were rigorously defined
- ✓ The actuator location optimization were applied to linear, nonlinear, and fluid problems and shown effective.
- ✓ The actuator location optimization were applied to weather modification problems
  - ✓ The randomized singular vector method efficiently produced results comparable to the standard implementation of singular vector method. The method was also applied to weather control problem in the framework of WRFPLUS.
  - ✓ Sensitivity analysis of surface water vapor to the accumulated precipitation of torrential rainfall was performed, and actuator positions that can be expected to actuate efficiently were obtained.
  - ✓ The obtained actuator positions were used to actuate in the tangent linear model with water vapor, and the intervention was confirmed to be more efficient than the random selection.

- Further investigation of control law
- Examination of the results of control experiments with nonlinear models