Google Quantum AI

Measurement-Induced State Transitions in Superconducting Qubits: Past, Present, and Future Considerations

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arXiv:2212.05097

Outline

- History of measurement-induced state transitions (MIST)

- Motivation: why are we still working on this problem?

- How we characterize MIST in experiment

- Observations in the $f_r < f_q$ regime:
    - Noisy time dependent behavior
    - Qubit reset following a state transition

- Basic theory of state transitions

History of measurement-induced state transitions (MIST)

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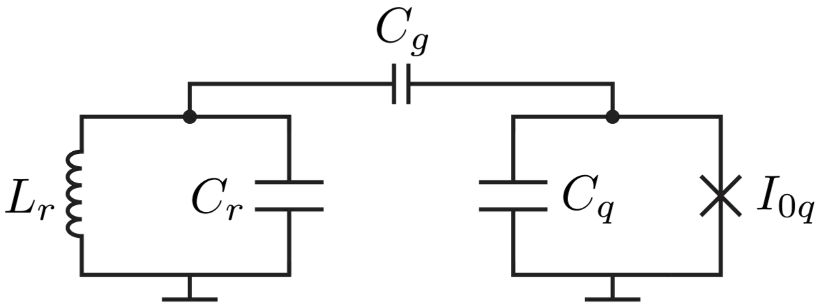
Dispersive Measurement 101



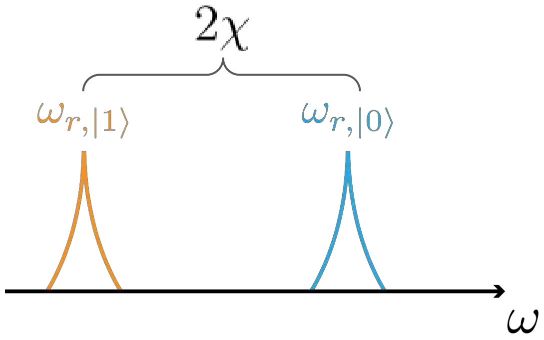
$$\hat{H}/\hbar = \omega_r \hat{a}^\dagger \hat{a} + \sum_j \omega_j |j\rangle\langle j| + \sum_{ij} g_{ij} |j\rangle\langle i| (\hat{a}^\dagger + \hat{a})$$

(RWA)

$$\approx \omega_r \hat{a}^\dagger \hat{a} + \sum_j \omega_j |j\rangle\langle j| + \sum_i g_{i,i+1} (|i\rangle\langle i+1|\ \hat{a}^\dagger + \mathrm{h.c.})$$

(TLS)

$$\approx \omega_r \hat{a}^\dagger \hat{a} + \frac{\omega_{10}}{2} \hat{\sigma}_z + g(\hat{a}^\dagger \hat{\sigma}_- + \hat{a} \hat{\sigma}_+)$$



(g<<Δ)

$$\Rightarrow \hat{H}_{\mathrm{eff}}/\hbar \approx (\omega_r - \chi \hat{\sigma}_z) \hat{a}^\dagger \hat{a} - \frac{\hat{\sigma}_z}{2} (\omega_{10} + \chi)$$

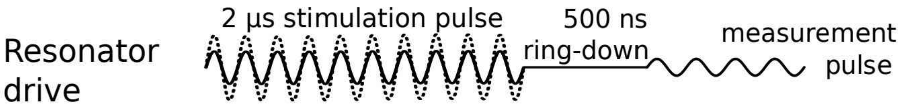
$$\chi = \frac{g^2}{\Delta},\ \Delta = \omega_{10} - \omega_r,\ g = \frac{\sqrt{\omega_r \omega_{10}}}{2} \frac{C_g}{\sqrt{C_r C_q}}$$

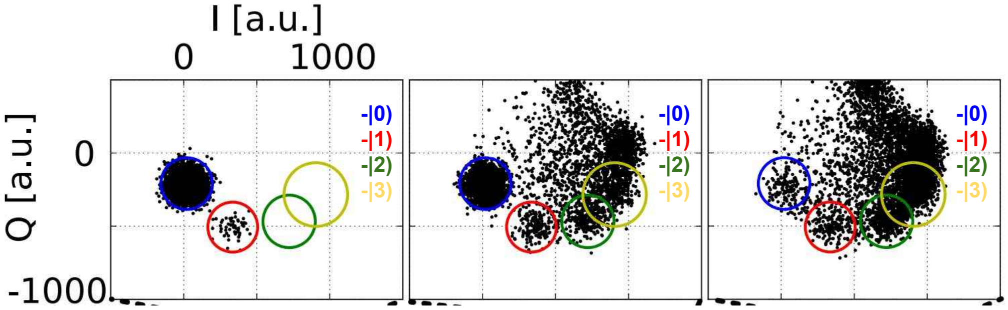
$$n_{\mathrm{crit}} = \frac{1}{4} \frac{|\Delta|^2}{g^2}$$

* E. Jaynes and F. Cummings *Proceedings of the IEEE,* 51, 89 (1963)
* M. Tavis and F. Cummings *Phys. Rev.* 170, 379 (1968)
* A. Wallraff et al. *Phys. Rev. Lett.*, 95, 060501 (2005)

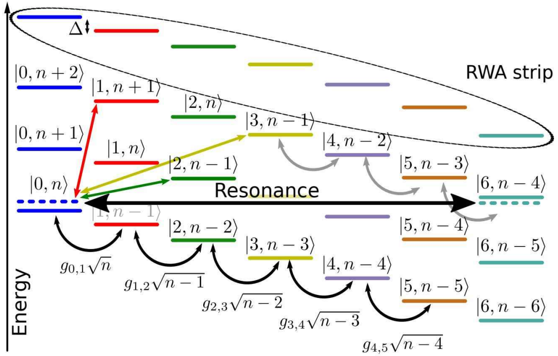
Non-RWA Terms Are Needed To Explain MIST

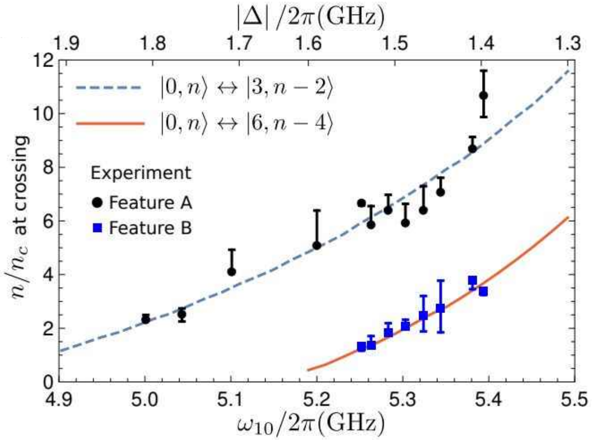
$f_r > f_q$





Crank up the resonator drive power





* D. Sank, M. Khezri, Z. Chen et al. *Phys. Rev. Lett.*, 117, 190503 (2016)

Motivation: Why Are We Still Working On This Problem?

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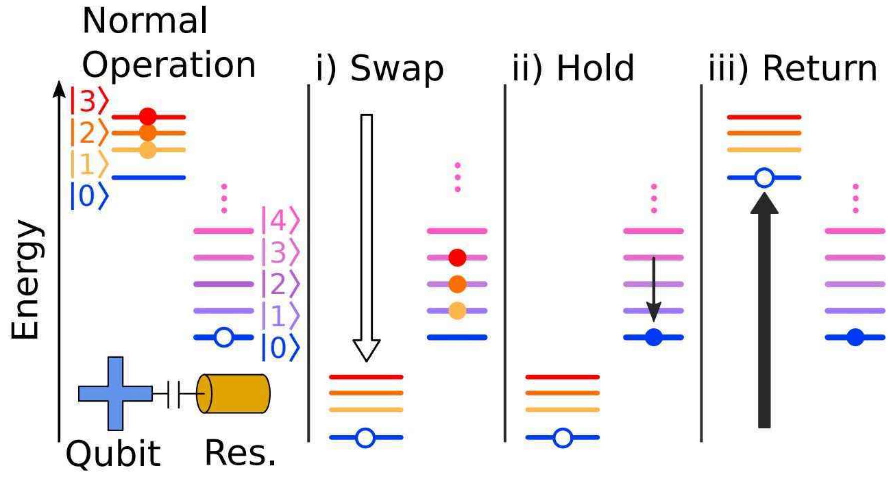
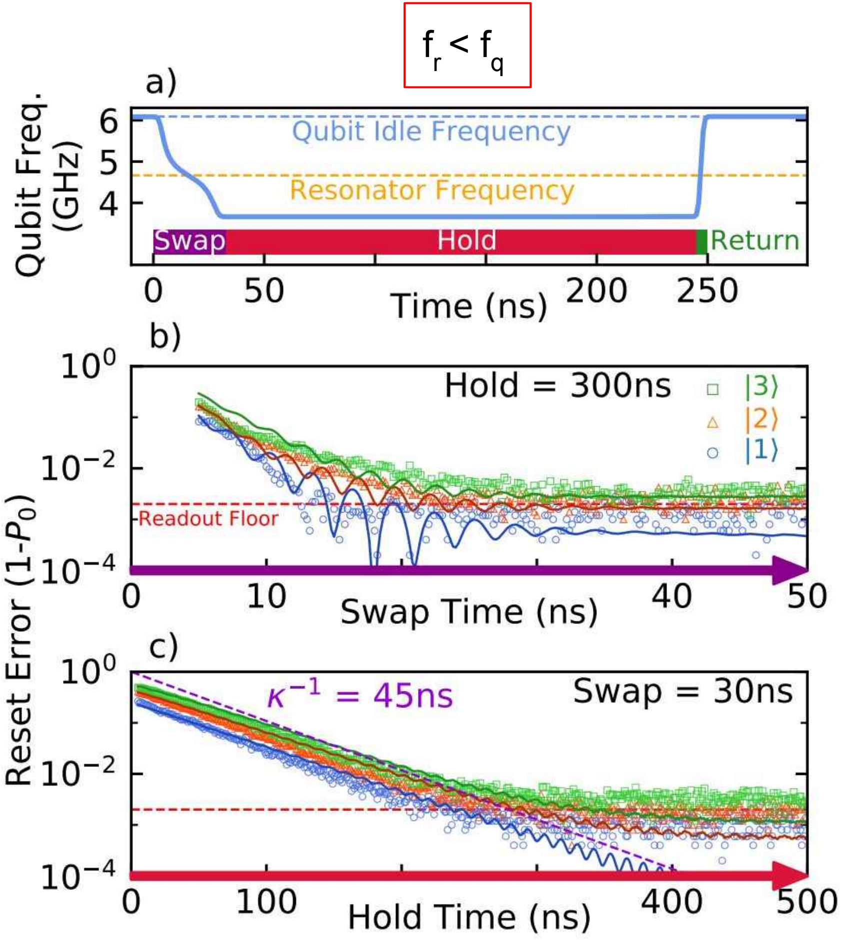
Error Correction Requires Qubit Reset





- Leakage builds up due to gates, heating, and readout ⇒ need to put them back in |0⟩
- <u>Solution</u>: bias qubit into resonance with lossy measurement resonators

*M. McEwen et al. *Nat. Commun.* 12, 1761 (2021)
*K. Miao, M. McEwen et al. arXiv:2211.04728 (2022)

How We Characterize State Transitions

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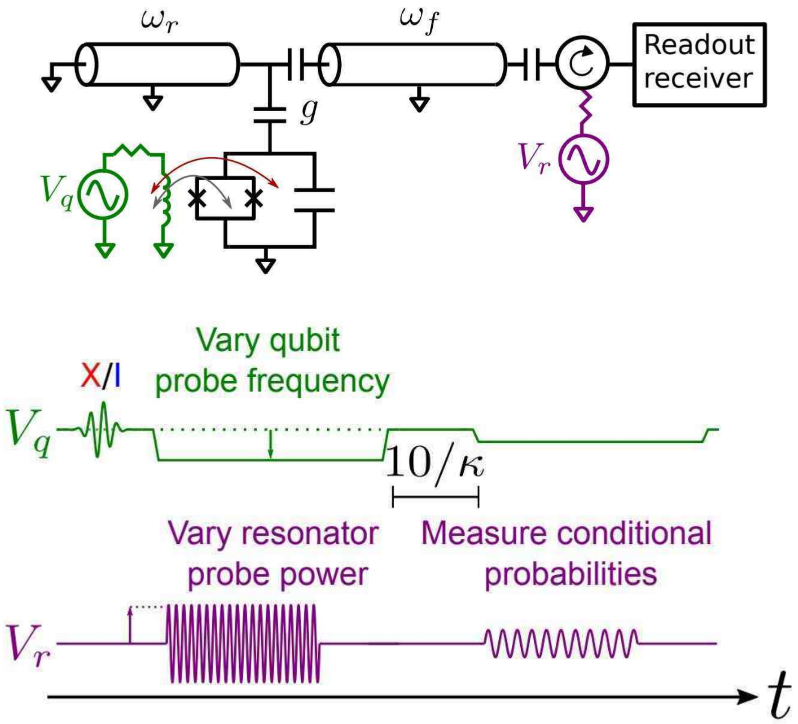
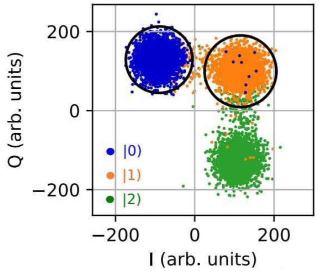
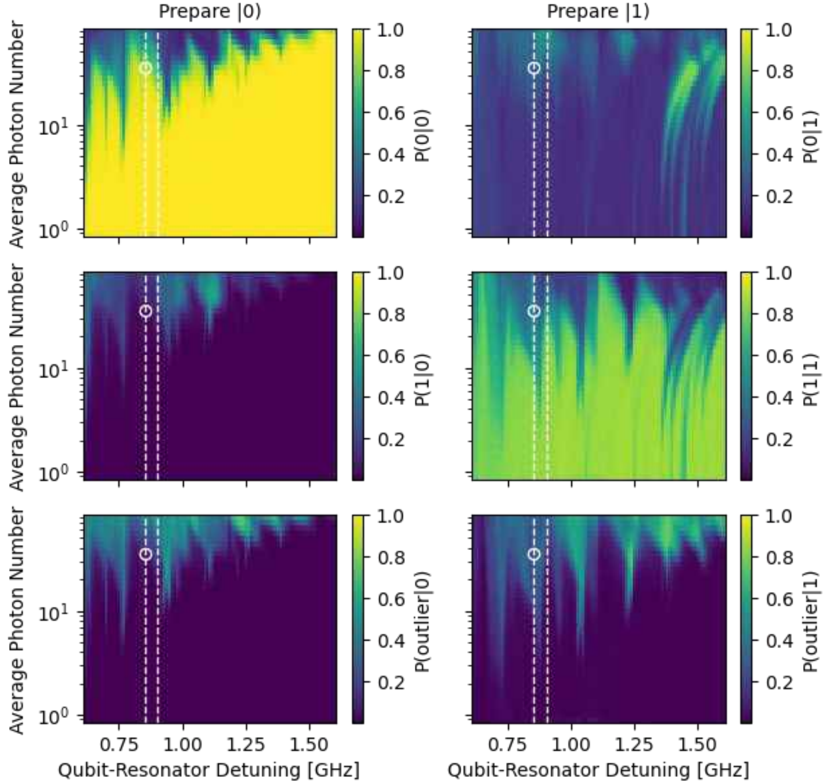
Experimental Procedure



$f_r < f_q$



outlier ≡ IQ shots outside of black circles



* D. Sank, M. Khezri, Z. Chen et al. *Phys. Rev. Lett.*, 117, 190503 (2016)
* M. Khezri, A. Opremcak et al. arXiv:2212.05097 (2022)

Experimental Observations

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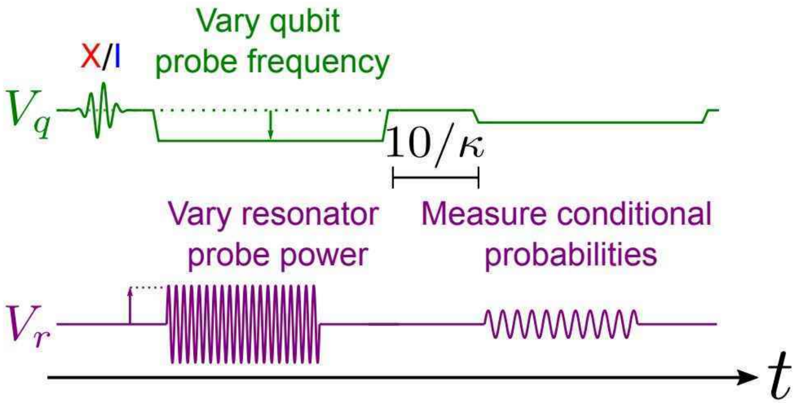
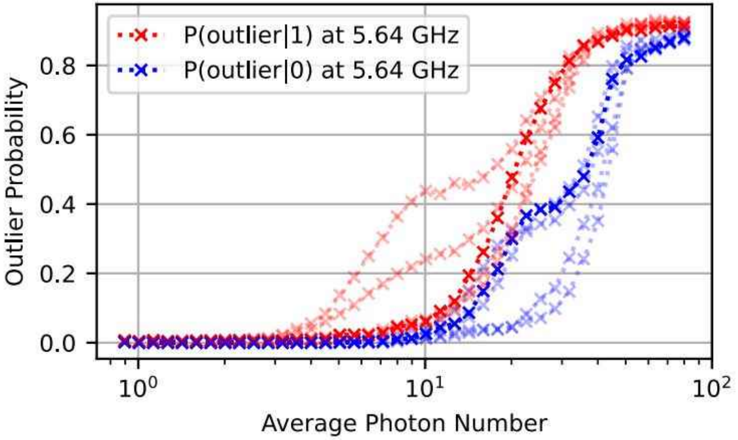
Time Dependence



$f_r < f_q$

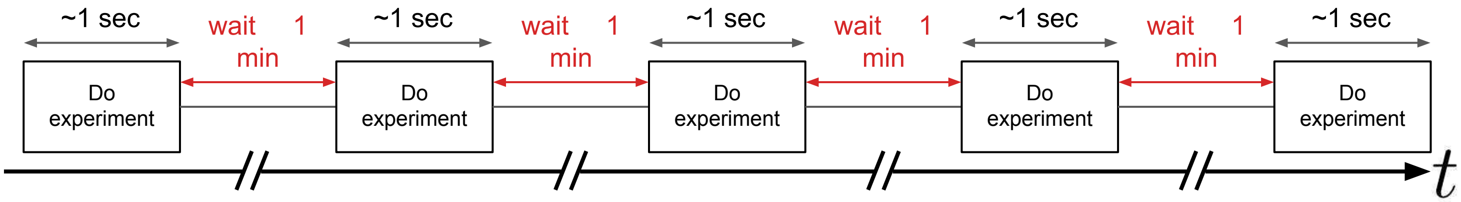


Experiment:

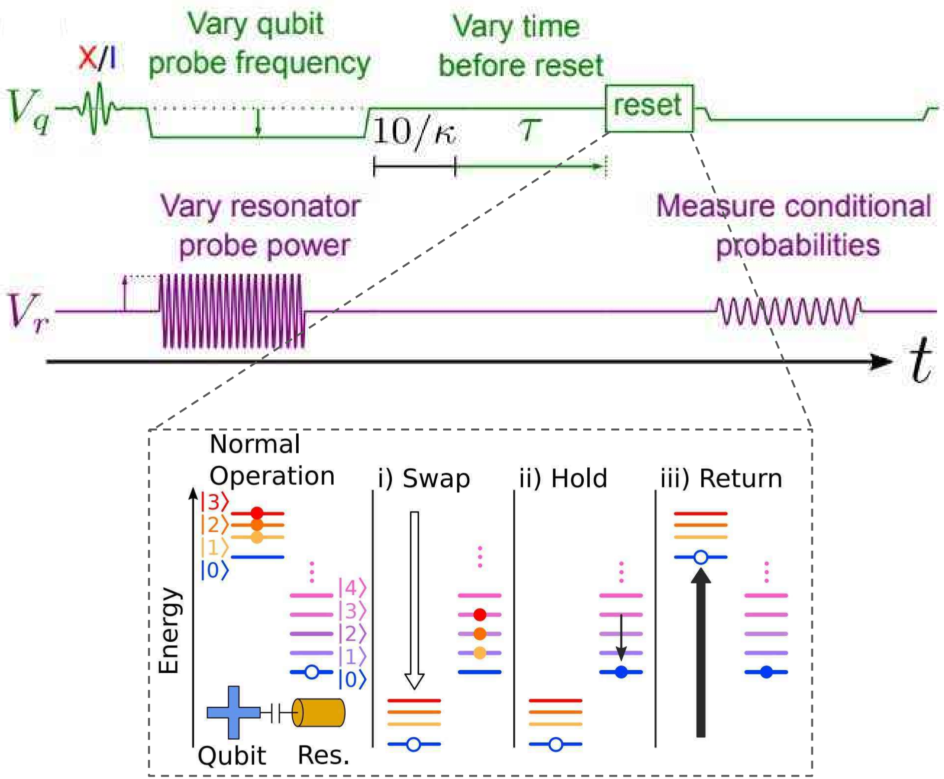
1. Prepare |0⟩ (|1⟩)
2. Bias qubit to 5.64 GHz
3. Sweep resonator probe power
4. Measurement conditional probabilities (5,000 stats/power)

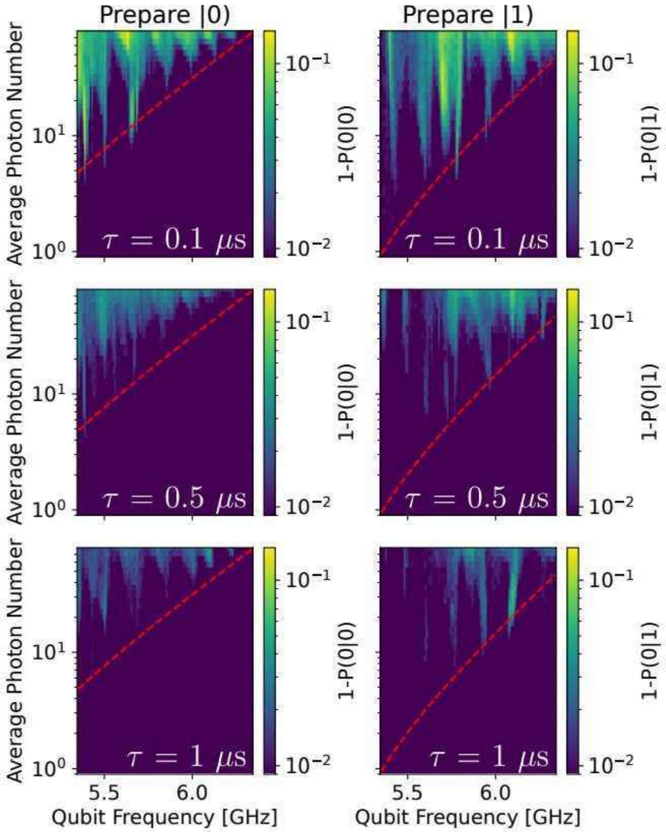
~1 sec to complete

- State transitions features change when measured repeatedly in time, on timescales ~ min



Qubit Reset Following a State Transition





*M. McEwen et al. *Nat. Commun.* 12, 1761 (2021)

- Measurement-induced leakage lives for microseconds after an event

Main Mechanism of readout transitions

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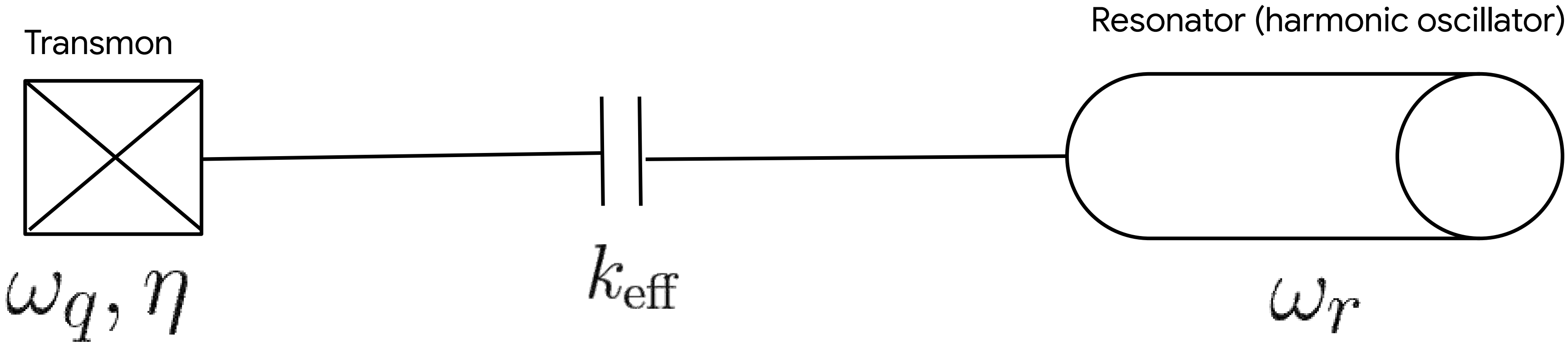
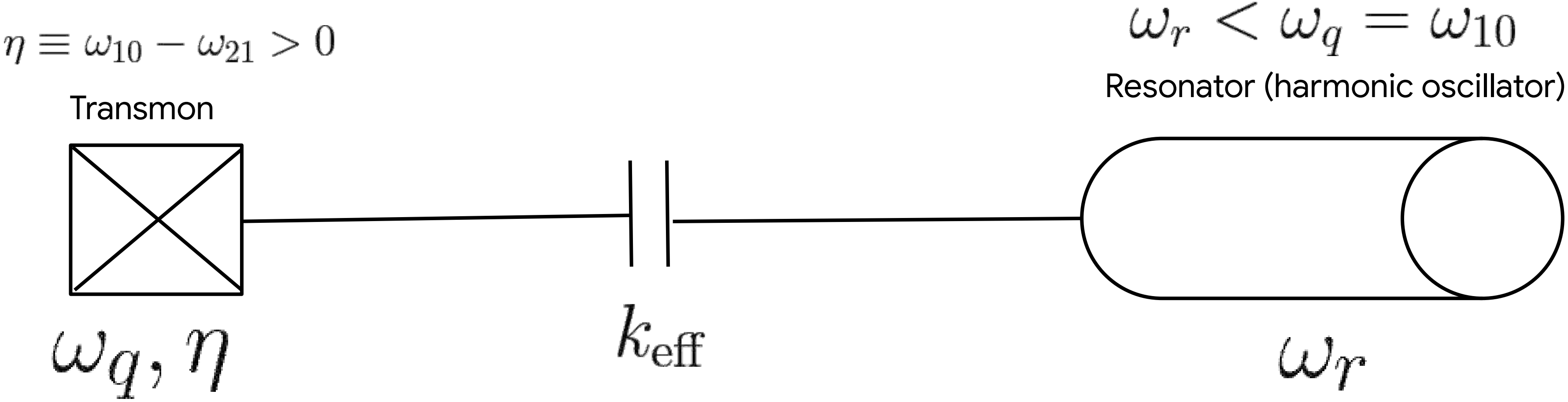
System

$$\eta \equiv \omega_{10} - \omega_{21} > 0$$

$$\omega_r < \omega_q = \omega_{10}$$



System



$$\underbrace{\sum_k E_k |k\rangle\langle k|}_{\substack{\text{Transmon eigensystem} \\ \text{depends on offset charge}}} + \underbrace{\sum_{k,k',n} g_{k,k'}\sqrt{n}|k', n-1\rangle\langle k, n| + \mathrm{H.c.}}_{\substack{\text{Charge-charge interaction} \\ \text{depends on offset charge}}} + \underbrace{\sum_n n\omega_r |n\rangle\langle n|}_{\text{resonator eigensystem}}$$

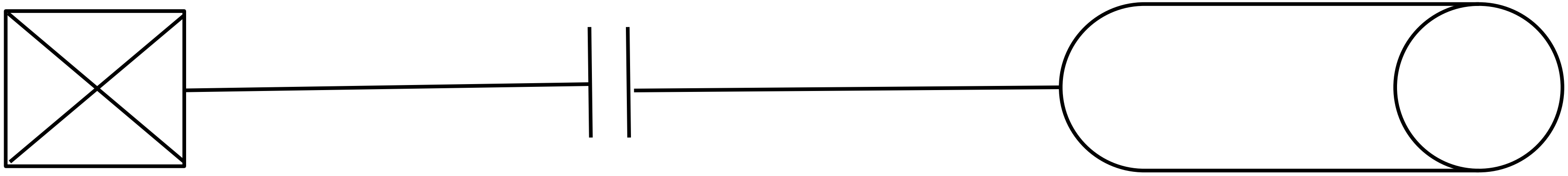
System

$\eta \equiv \omega_{10} - \omega_{21} > 0$

$\omega_r < \omega_q = \omega_{10}$

Transmon

Resonator (harmonic oscillator)



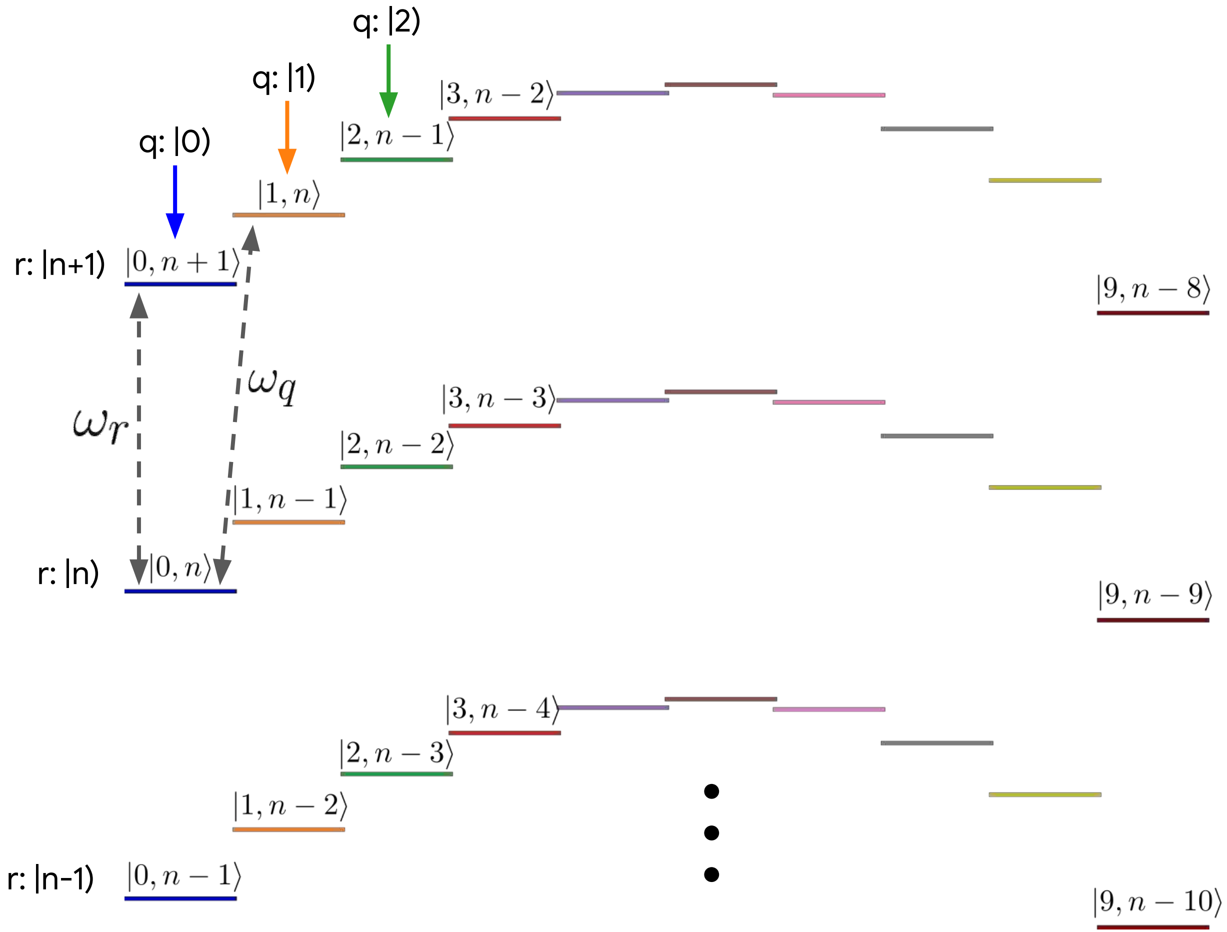
ω_q, η $\quad k_{\text{eff}}$ $\quad \omega_r$

$$\sum_k E_k |k\rangle\langle k| + \sum_{k,k',n} g_{k,k'} \sqrt{n} |k', n-1\rangle\langle k, n| + \text{H.c.} + \sum_n n\omega_r |n\rangle\langle n|$$

Transmon charge matrix elements: $g_{k,k'} = \frac{1}{2} k_{\text{eff}} \sqrt{\omega_r \omega_q} \frac{\langle k|\hat{n}_q|k'\rangle}{\langle 0|\hat{n}_q|1\rangle}$

Jaynes-Tavis-Cummings ladder

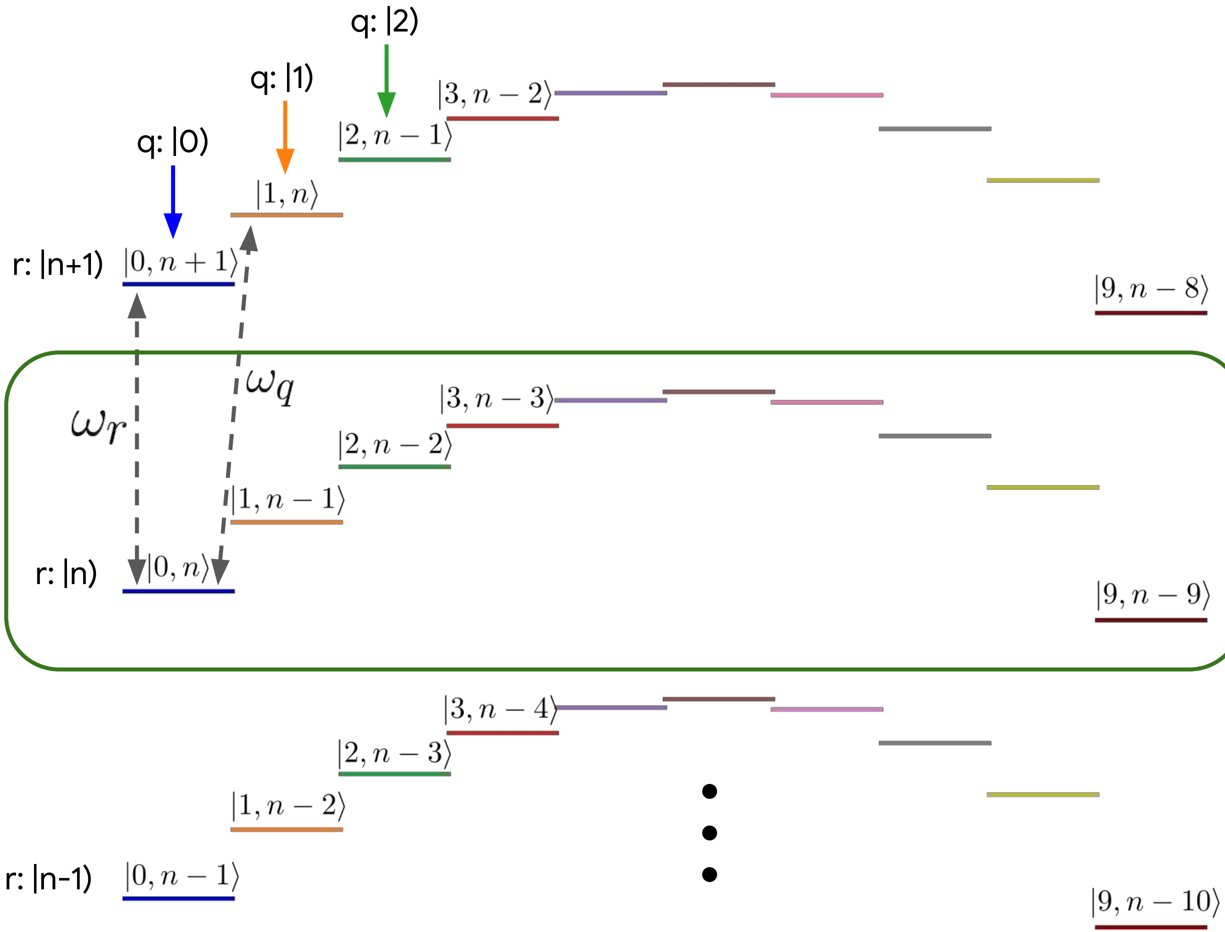
- Energy ladder of qubit and resonator
- Resonator below the qubit in frequency



$|\text{qubit}, \text{resonator}\rangle$

Jaynes-Tavis-Cummings ladder

- Energy ladder of qubit and resonator
- Resonator below the qubit
- **RWA strip** is a set of energy levels where total number of excitations is fixed
  - Rotating wave approximation

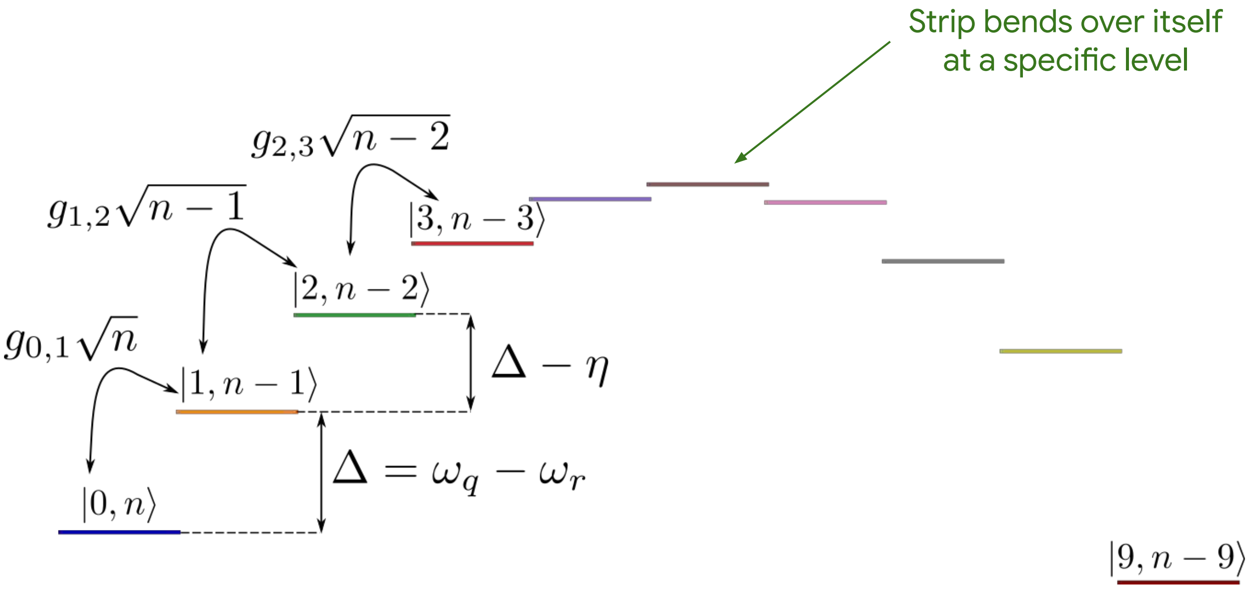


RWA strip

- Bare energy levels in RWA strip are coupled together via excitation preserving interactions

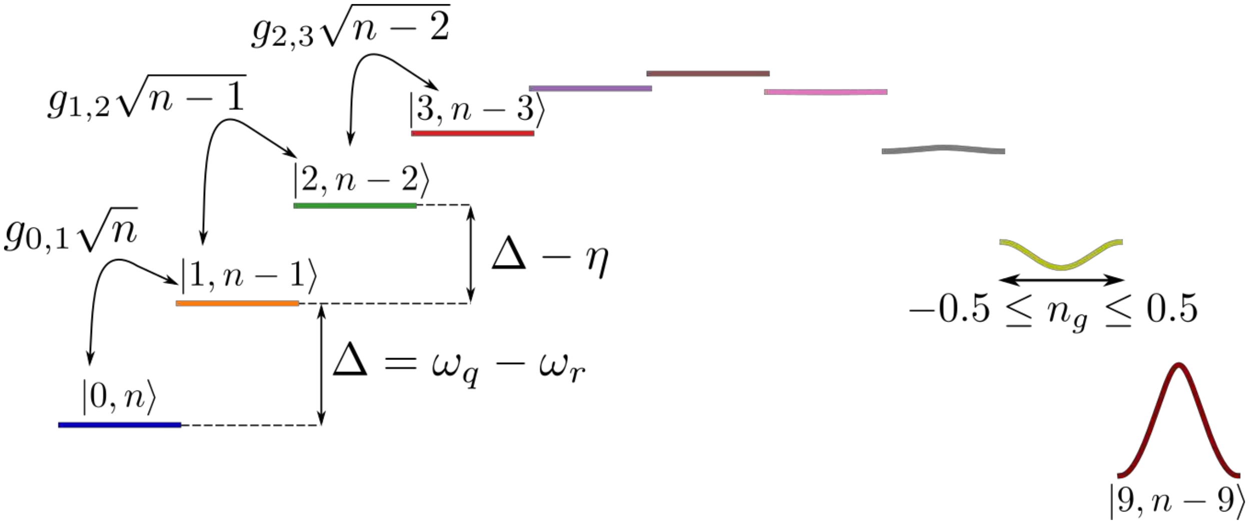
- Interaction strength depends on qubit charge matrix and resonator photons

- RWA strip bends over itself at a specific level $k_{\mathrm{b}} \approx \Delta/\eta$



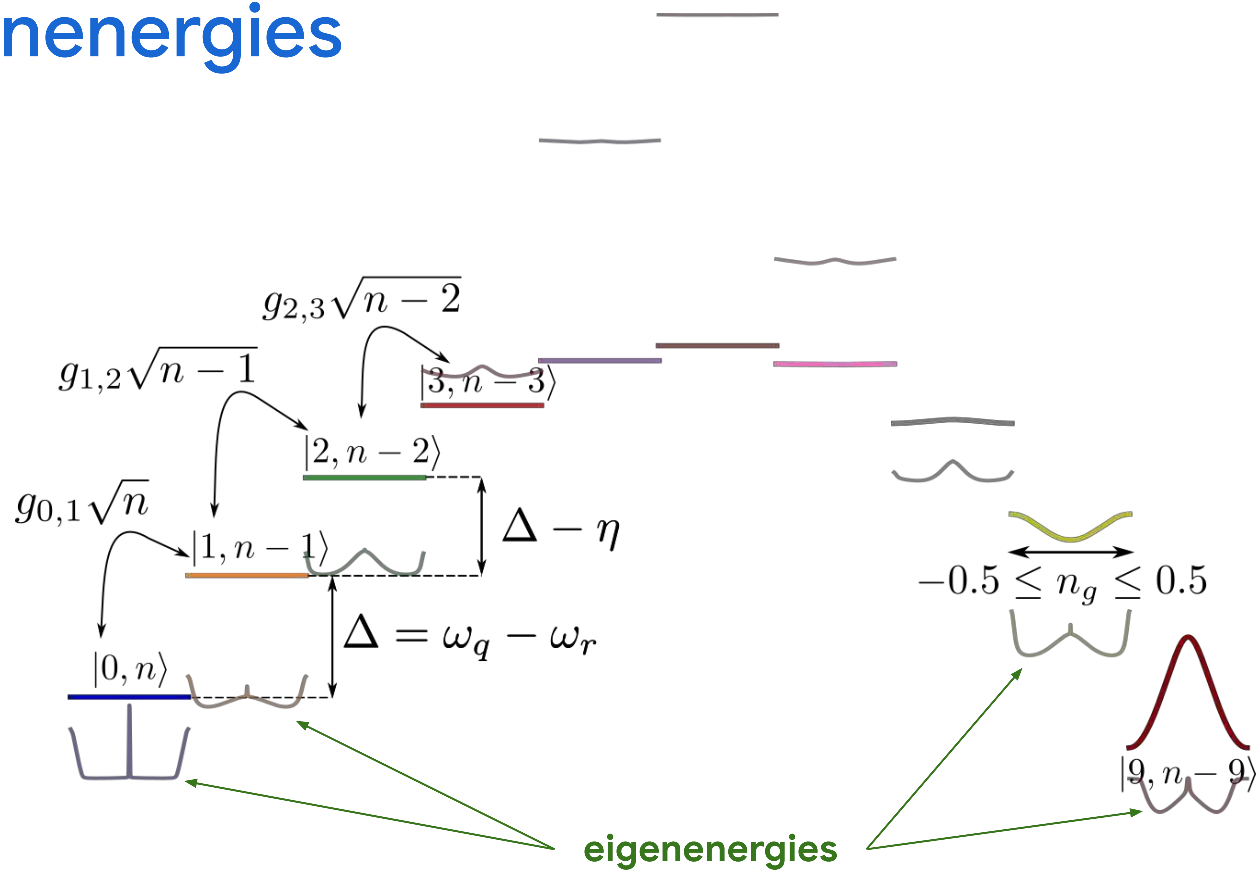
RWA strip with offset charge

- Energy of higher transmon levels depends on the offset charge
- Offset charge is scanned along the length of each energy level



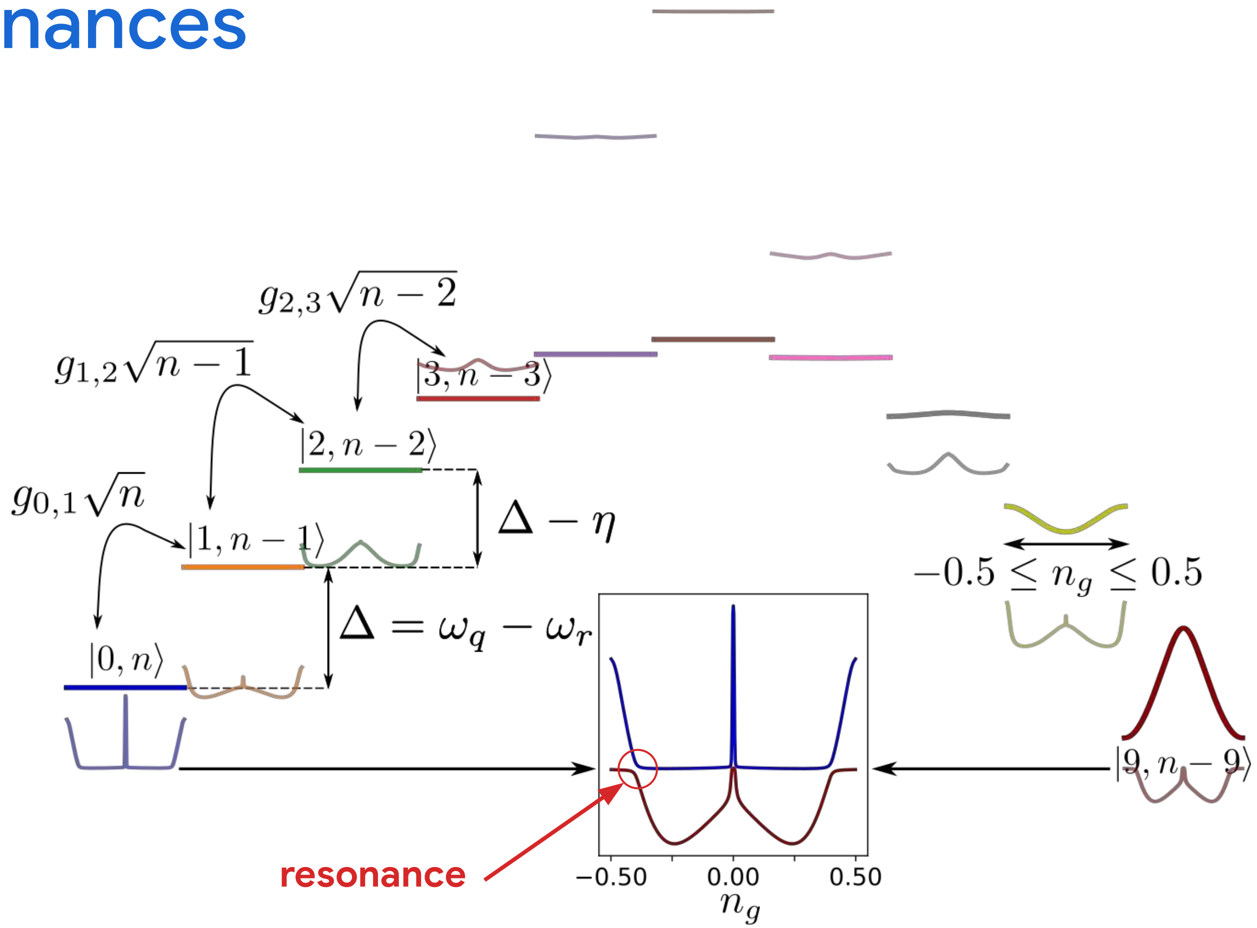
RWA strip eigenenergies

- Resonator photons couple level together to form eigenenergies
  - Thin lines in diagram
- Eigenenergies depend on the offset charge, even for lower transmon levels
  - Similar observation by Cohen et al., arXiv:2207.09361
- Eigenenergy formation strongly influenced by where the strip bends over itself, i.e., detuning



RWA strip resonances

- Eigenenergy resonances occur within RWA strip between lower and higher levels
    - Avoided crossing
    - Qubit population can move between resonant levels via Landau-Zener
- Resonance condition depends on photon number *and* offset charge



Model for predicting onset of readout transitions

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System Hamiltonian

$$
\begin{aligned}
H =& 4E_C(\hat{n} - n_g)^2 - E_J \cos(\hat{\varphi}) \\
& + \omega_r \hat{a}^\dagger \hat{a} + \varepsilon e^{-i\omega_d t} \hat{a}^\dagger + \varepsilon^* e^{i\omega_d t} \hat{a} \\
& + g' \hat{n} (\hat{a} - \hat{a}^\dagger)
\end{aligned}
$$

Transmon circuit

Resonator and its drive (RWA drive)

charge-charge interaction

RWA System Hamiltonian in the rotating frame

Transmon eigenenergies

$$H^{\mathrm{RWA}}_{\mathrm{r.f.}} = \sum_{k,n} (E_k - k\omega_r)|k,n\rangle\langle k,n|$$

RWA strip energies

$$+ (\omega_r - \omega_d)\hat{a}^\dagger\hat{a} + \varepsilon\hat{a}^\dagger + \varepsilon^*\hat{a}$$

Resonator and its drive (RWA drive)

$$+ \hat{a}^\dagger e^{-i(\omega_r-\omega_d)t} \sum_{k,n} g_{k,k+1}|k,n\rangle\langle k+1,n| + \mathrm{H.c.}$$

Excitation preserving interactions (RWA)

- Use ideology of dressed coherent state (arXiv:1606.04204)
  - In dispersive readout, system can be described as a coherent state made of joint eigenstates of qubit-resonator

- Treat resonator as a classical coherent state, which also drives the qubit
- Qubit becomes dressed due to interaction with resonator

$$a = \sum_{k,n} \sqrt{n}|k,n-1\rangle\langle k,n|$$

Direct drive of the transmon (main model)

$$H_{\text{eff}} = \sum_k (E_k - k\omega_r)|k\rangle\langle k|$$

RWA strip energies

$$+ \alpha^*(t) e^{-i(\omega_r - \omega_d)t} \sum_k g_{k,k+1}|k\rangle\langle k+1| + \text{H.c.},$$

Direct drive interaction

$$\alpha^* = \langle \hat{a}^\dagger \rangle$$

$$\dot{\alpha}(t) = -i(\omega_r - \omega_d)\alpha(t) - \kappa/2\,\alpha(t) - i\varepsilon$$

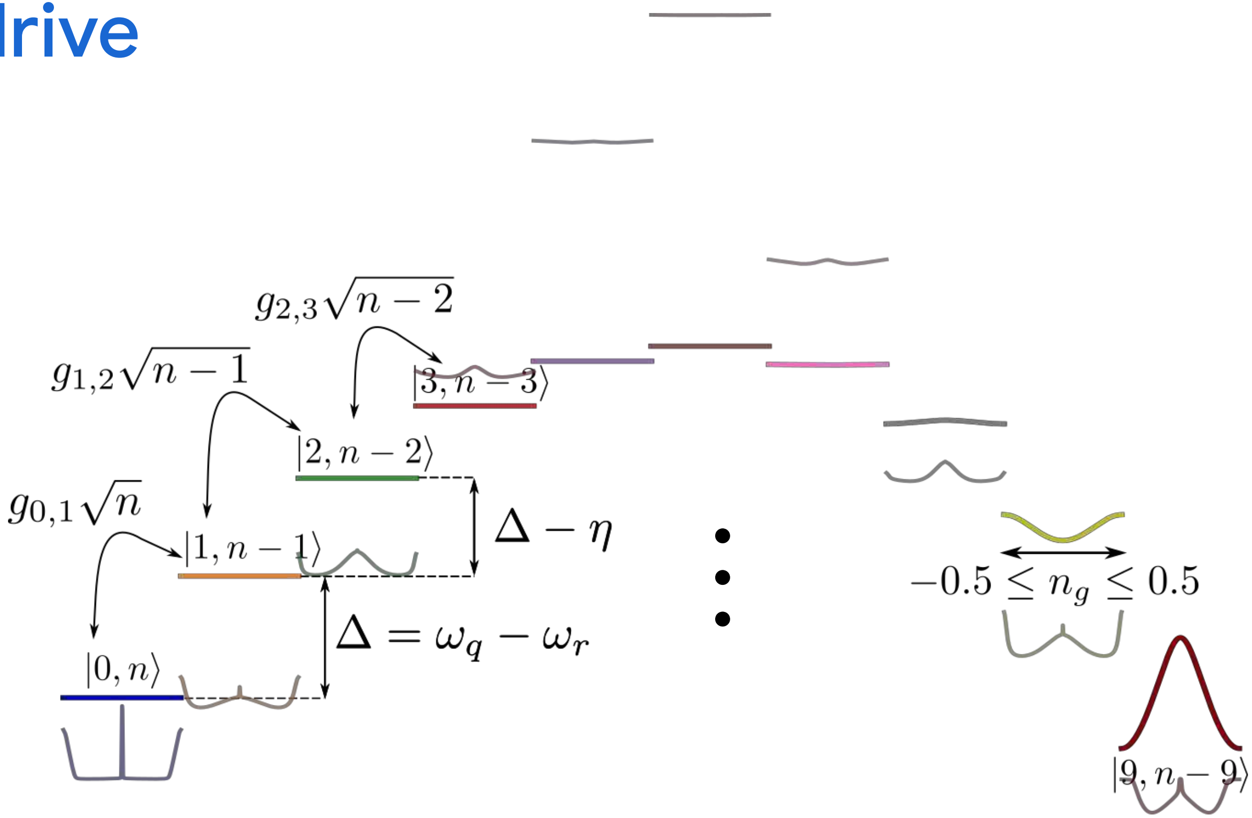
Coherent state evolution

$$\underbrace{\bar{n} = |\alpha|^2 \pm |\alpha|}_{\text{Photon number uncertainty in this model}}$$

- Resonator evolves according to coherent state evolution
    - Decoupled from qubit, assumption valid when qubit state hasn't changed
- Resonator is a coherent field directly driving the transmon, causing evolution in the RWA strip

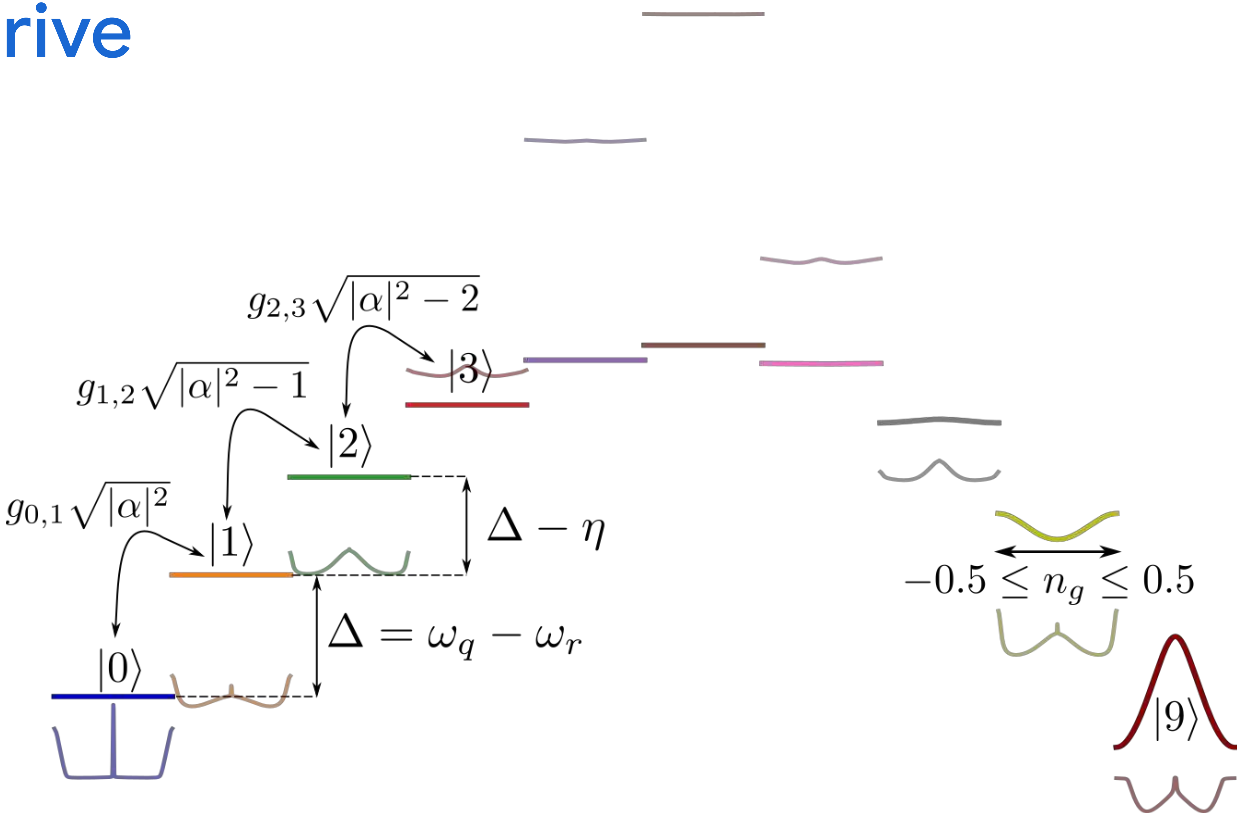
Idea of direct drive

- Full JTC ladder includes many copies of RWA strips
  - Each copy is slightly different

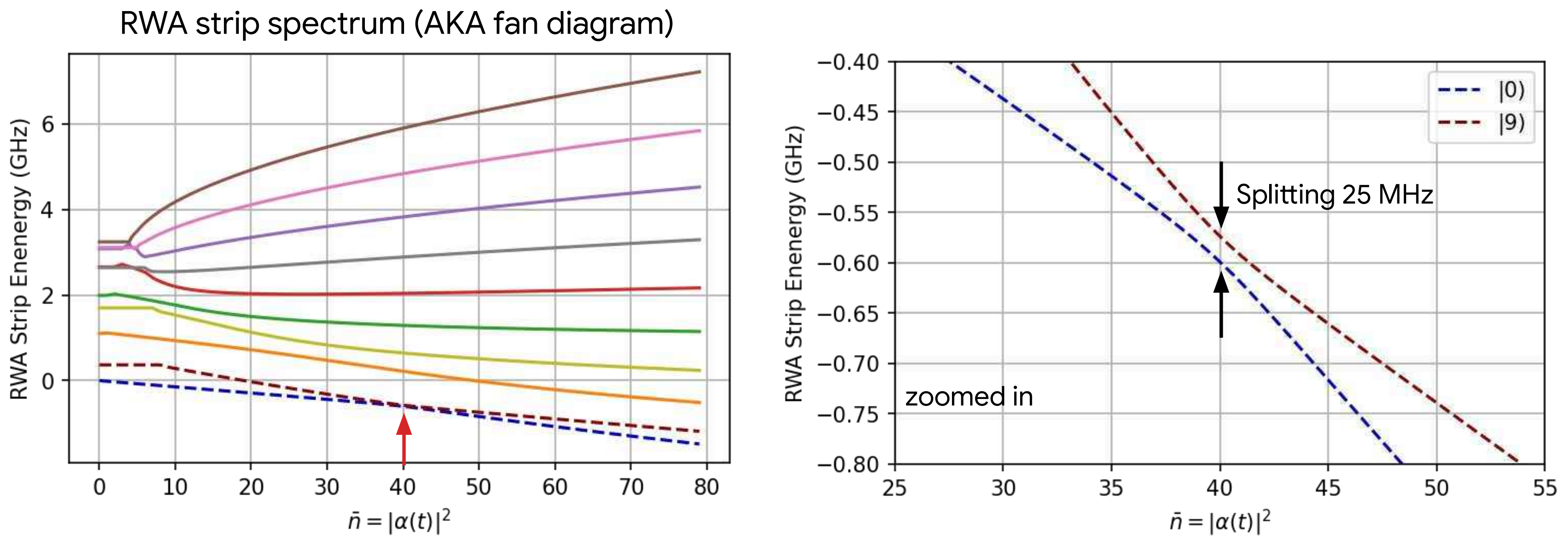


Idea of direct drive

- Direct drive focuses on one RWA strip
- Direct drive interactions mimic the JTC interactions
- Coherent state evolution ramps up the interactions within the RWA strip



Example of model results

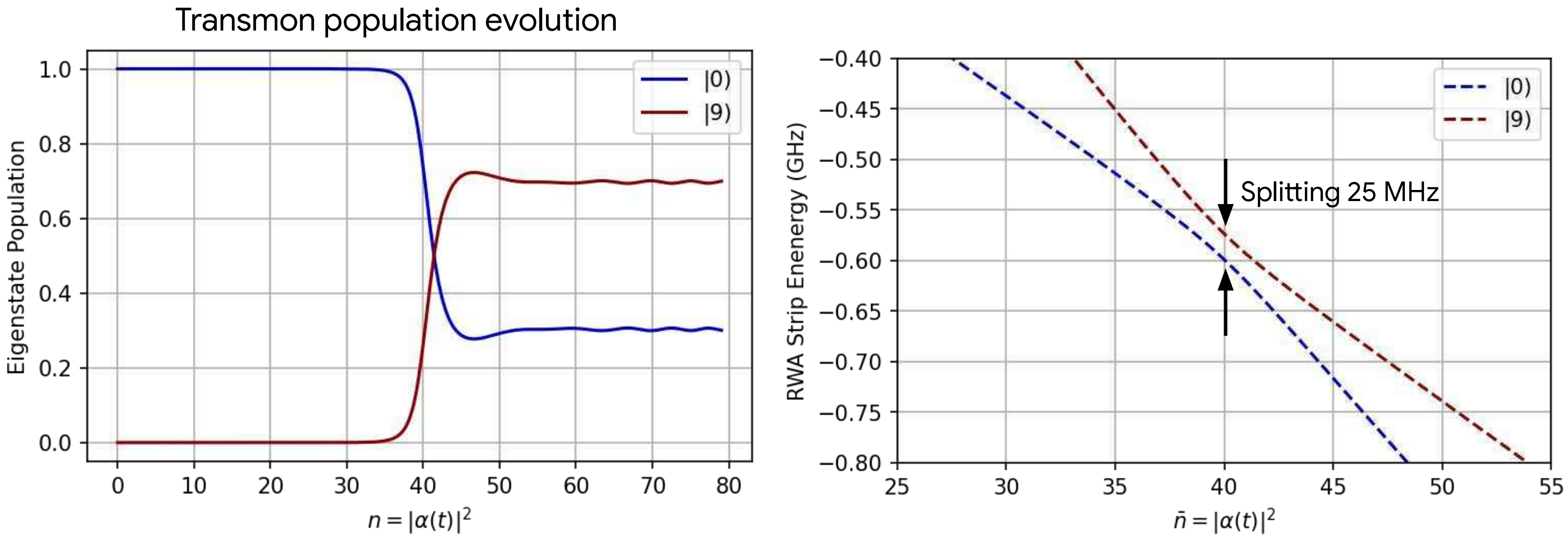


- Spectrum of the RWA strip indicates avoided crossing between levels
- This example shows avoided crossing between |0⟩ and |9⟩ at 40 photons

$n_g = 0.2$

$\Delta = 1.1\ \mathrm{GHz}$

Example of model results



- At the avoided crossing, qubit population is exchanged between levels via LZ
- Loss of initial qubit population signifies a measurement induced transition
- This all depend on qubit frequency and offset charge

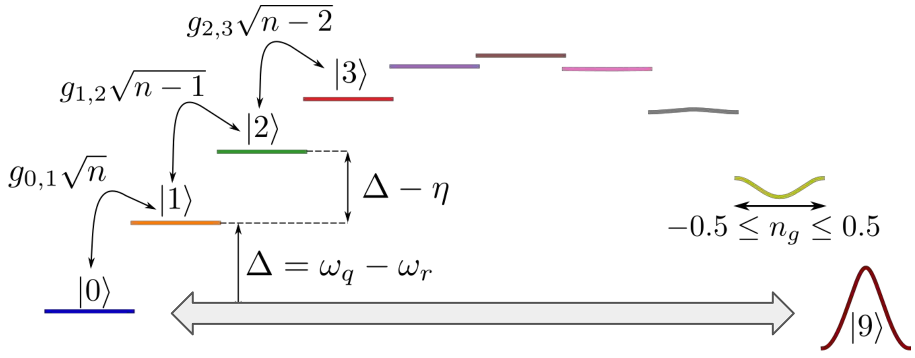
$n_g = 0.2$

$\Delta = 1.1\ \text{GHz}$

Perturbative effective coupling

$$g_{\text{eff}} \approx \frac{g_{0,1}g_{1,2}\cdots g_{8,9}}{\Delta_{1,0}\Delta_{2,0}\cdots\Delta_{8,0}}\bar{n}_{\text{cross}}^{9/2} \qquad g_{\text{eff}} \propto \left(\frac{\bar{n}_{\text{cross}}}{n_{\text{crit}}}\right)^4 g\sqrt{\bar{n}_{\text{cross}}}$$

- Effective coupling between |0⟩ and |9⟩ mediated via detuned “virtual levels” in between them
- Perturbative formula gives similar magnitude compared to simulation (*34 MHz vs. 46 MHz*)
    - No significant suppression due to detuned levels in between
- We typically need n comparable to critical photon number to have large enough effective coupling between levels



$$\omega_q = 5.735 \text{ GHz}$$
$$n_g = 0.1$$

$$n_{\text{crit}} \equiv (\Delta/g)^2/4 \approx 20$$
$$n_{\text{cross}} = 45$$

Model vs. experiments

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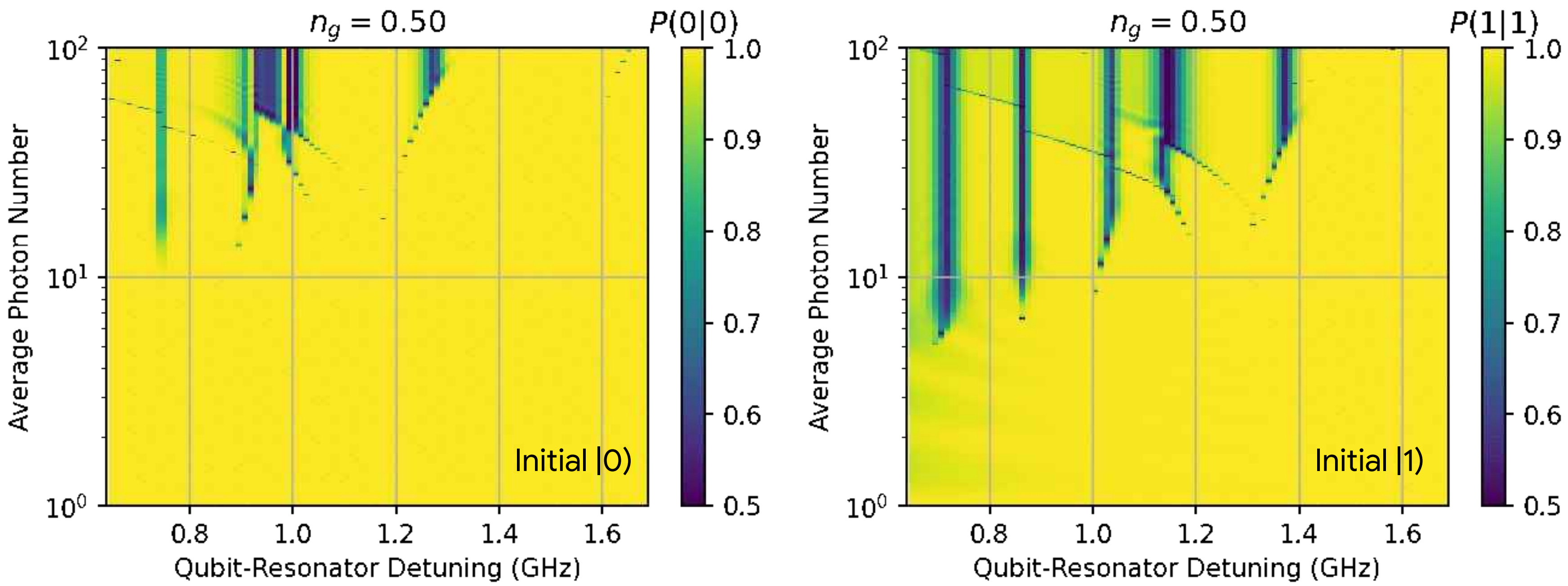
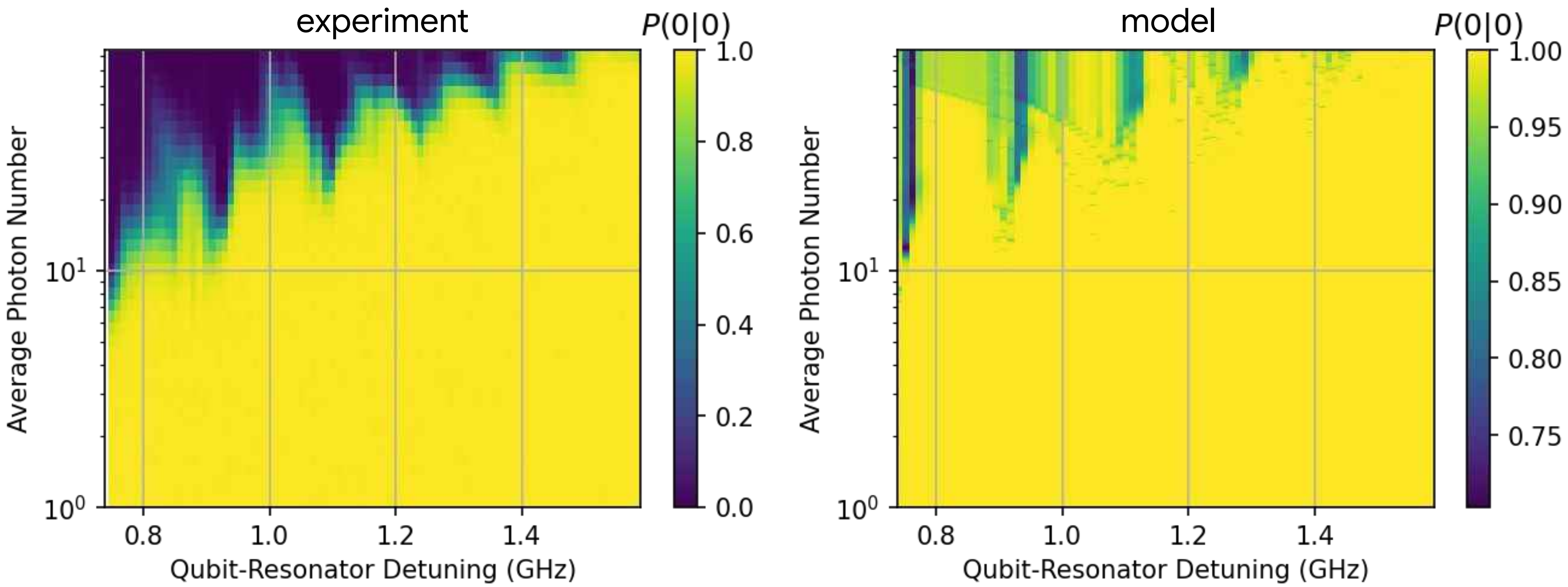
Transitions and changing background charge



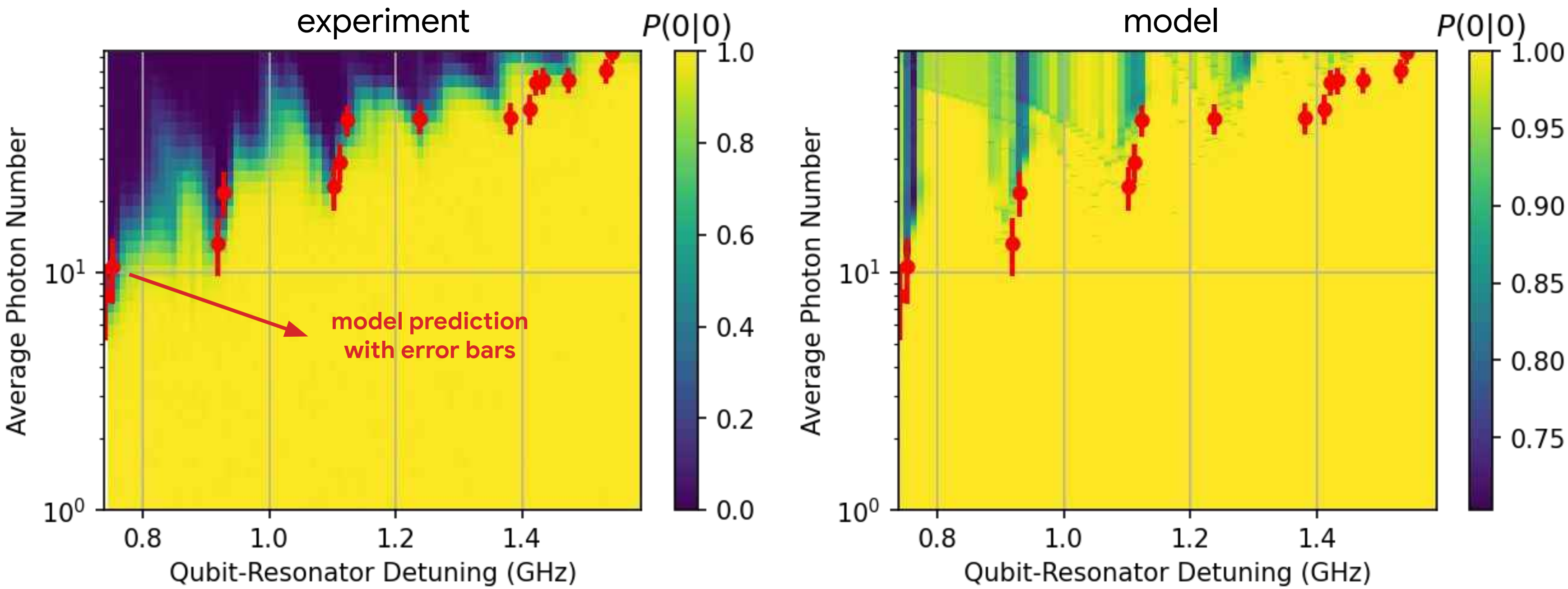
- Transitions are different for each value of offset charge, need to average over them
- Transitions occur around specific bands of resonance
  - Grouping of resonances due to bending of RWA strip

Model vs. experiment: initial $|0\rangle$



- Good qualitative and quantitative agreement between experiment and model for predicting onset of transitions
    - Model uses experimentally extracted parameters
- Model has an intrinsic $\pm\sqrt{n}$ uncertainty

Model vs. experiment: initial $|0\rangle$



- Good qualitative and quantitative agreement between experiment and model for predicting onset of transitions
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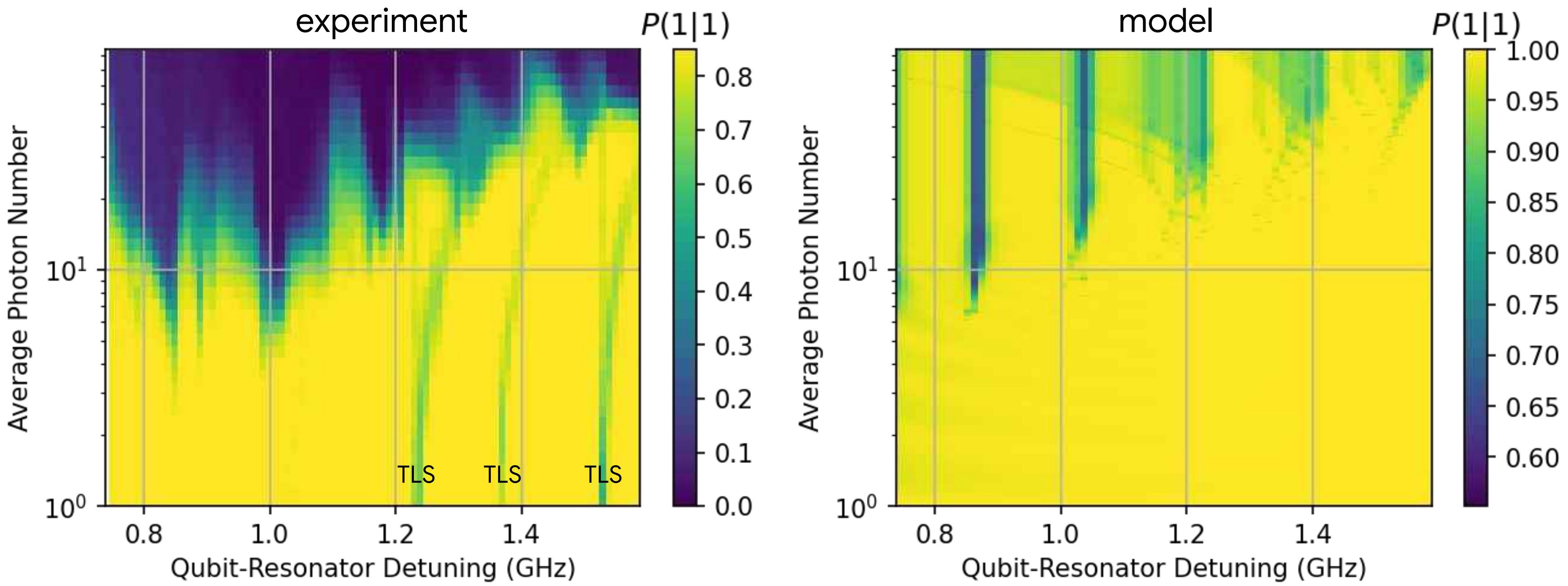
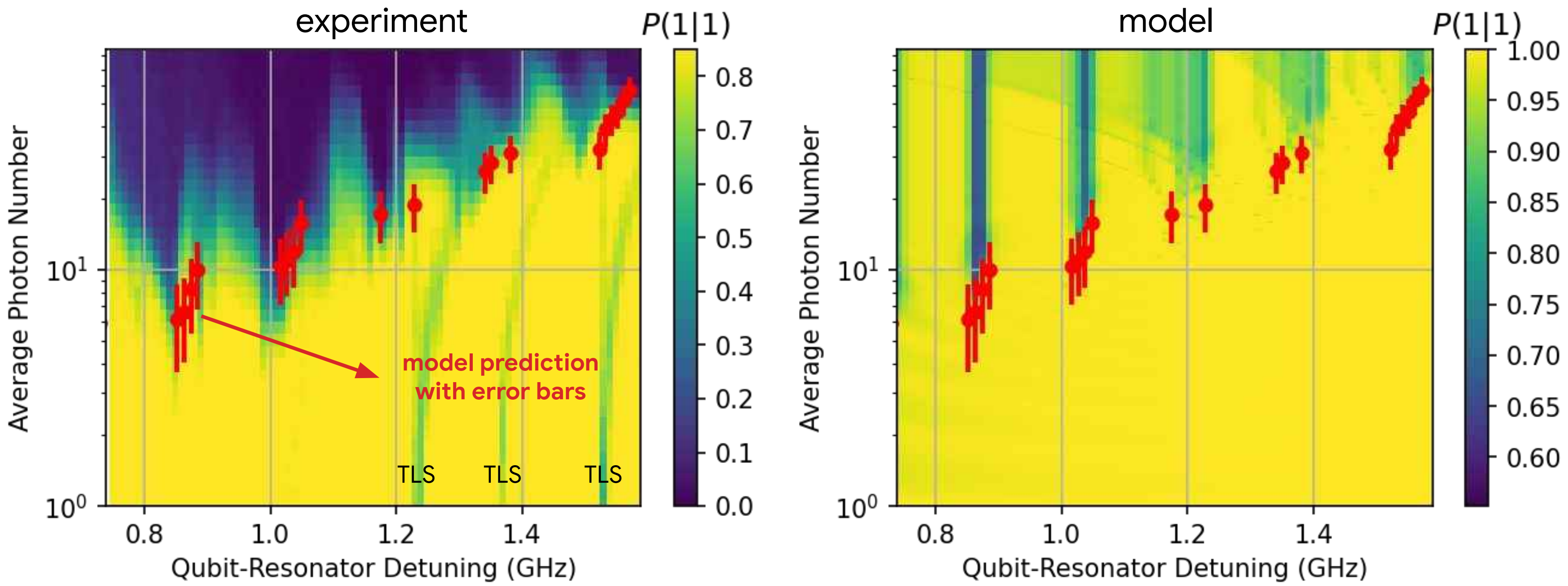
Model vs. experiment: initial $|1\rangle$



- Good qualitative and quantitative agreement between experiment and model for predicting onset of transitions
    - Model uses experimentally extracted parameters
- Model has an intrinsic $\pm\sqrt{n}$ uncertainty

Model vs. experiment: initial $|1\rangle$



- Good qualitative and quantitative agreement between experiment and model for predicting onset of transitions
  - Model uses experimentally extracted parameters
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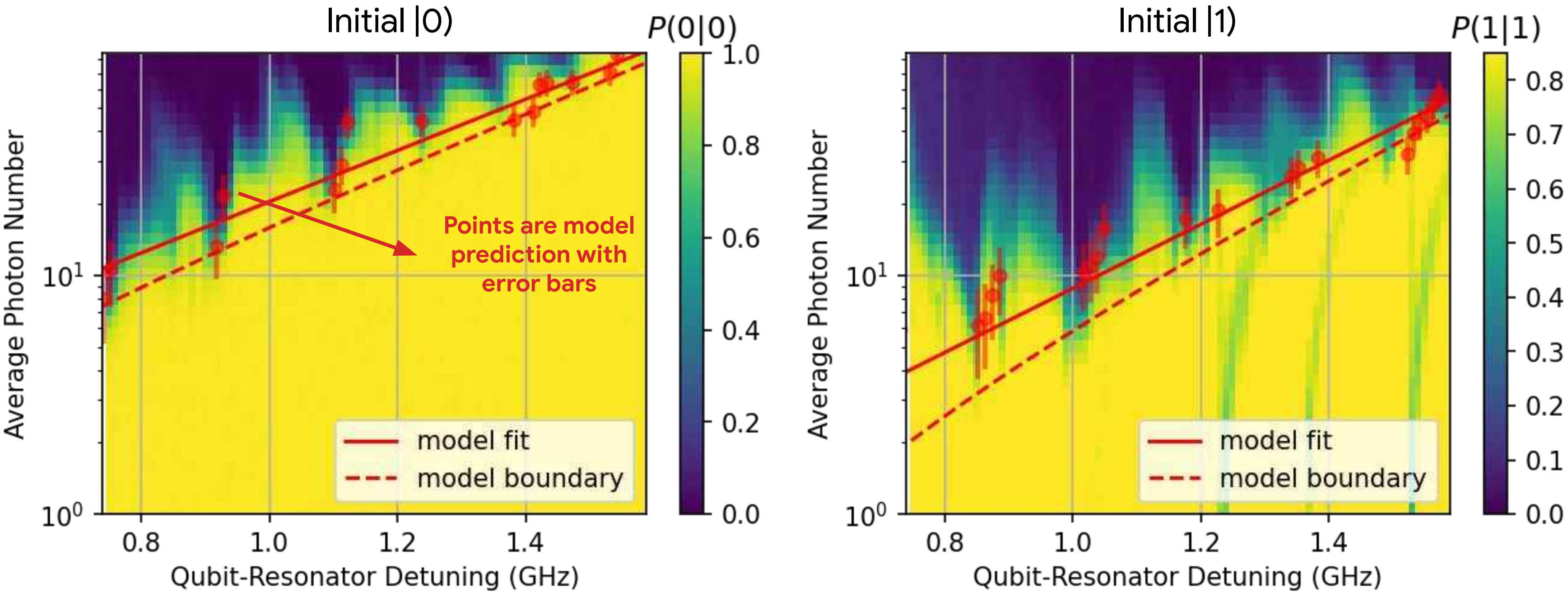
Boundary of readout transitions

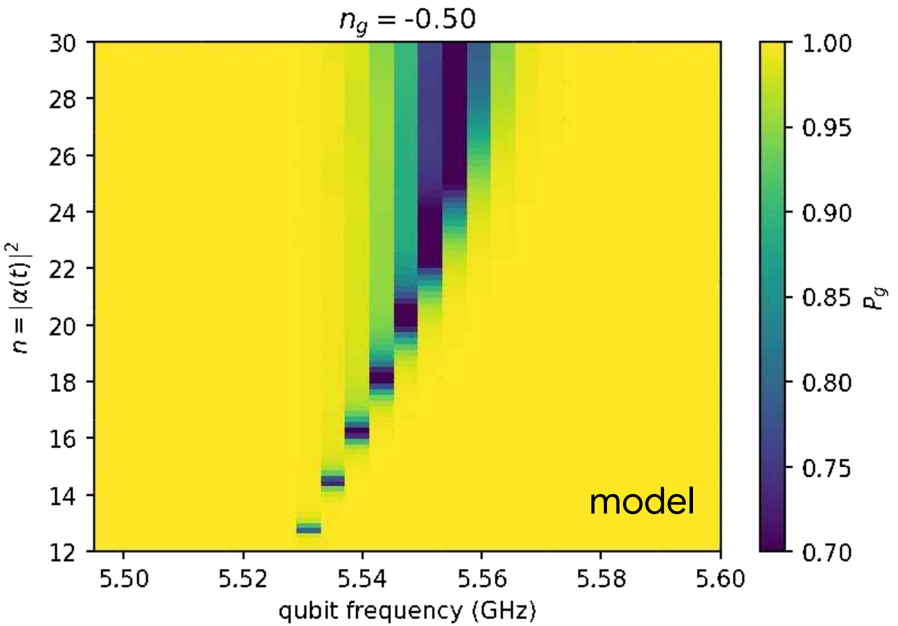


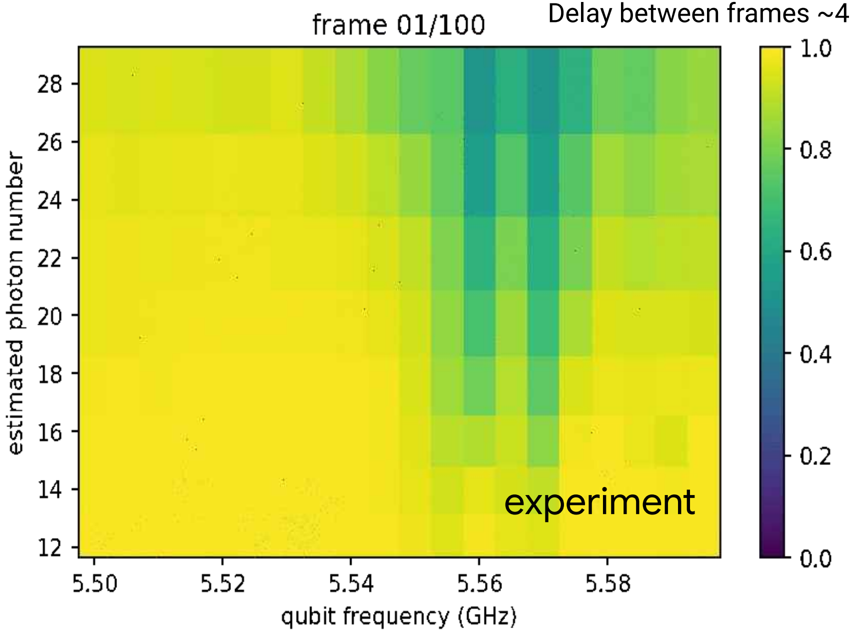
- Points are model prediction with $\pm\sqrt{n}$ error bars overlaid on experiment
- Solid line is an exponential fit to the model points $n_{\text{fit}} = A\exp(B\Delta)$
- Dashed line is transition boundary defined as $n_{\text{fit}} - \sqrt{n_{\text{fit}}}$

Change of offset charge in time

Each scan ~4 s

Delay between frames ~4 s





- Background charge is noisy and changes in time, and changes location of resonance bands
- Same behavior is observed in both the model and experiment

Initial |0⟩

Summary

- High-power readout will excite the qubit outside of the computational subspace
  - Limits use of power to speed up the readout

- This is caused by resonances within the energy ladder of qubit-resonator system
  - For resonator below the qubit in frequency, RWA is dominant

- We have developed a semi-classical models that can predict the occurrence of readout transitions in frequency and photon number

- We have validated our model by experimental data on multiple devices