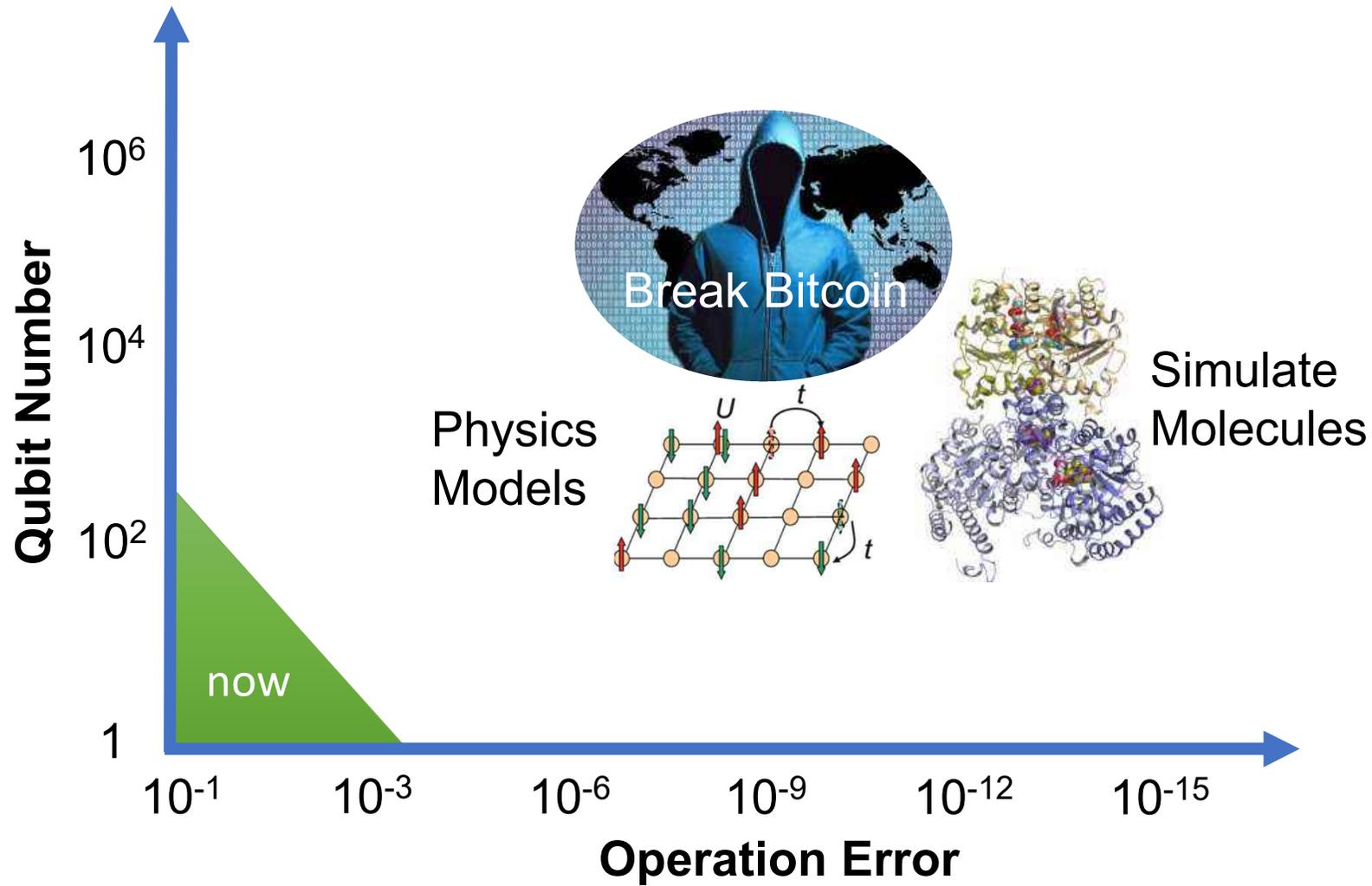


# Quantum Error Correction Now!

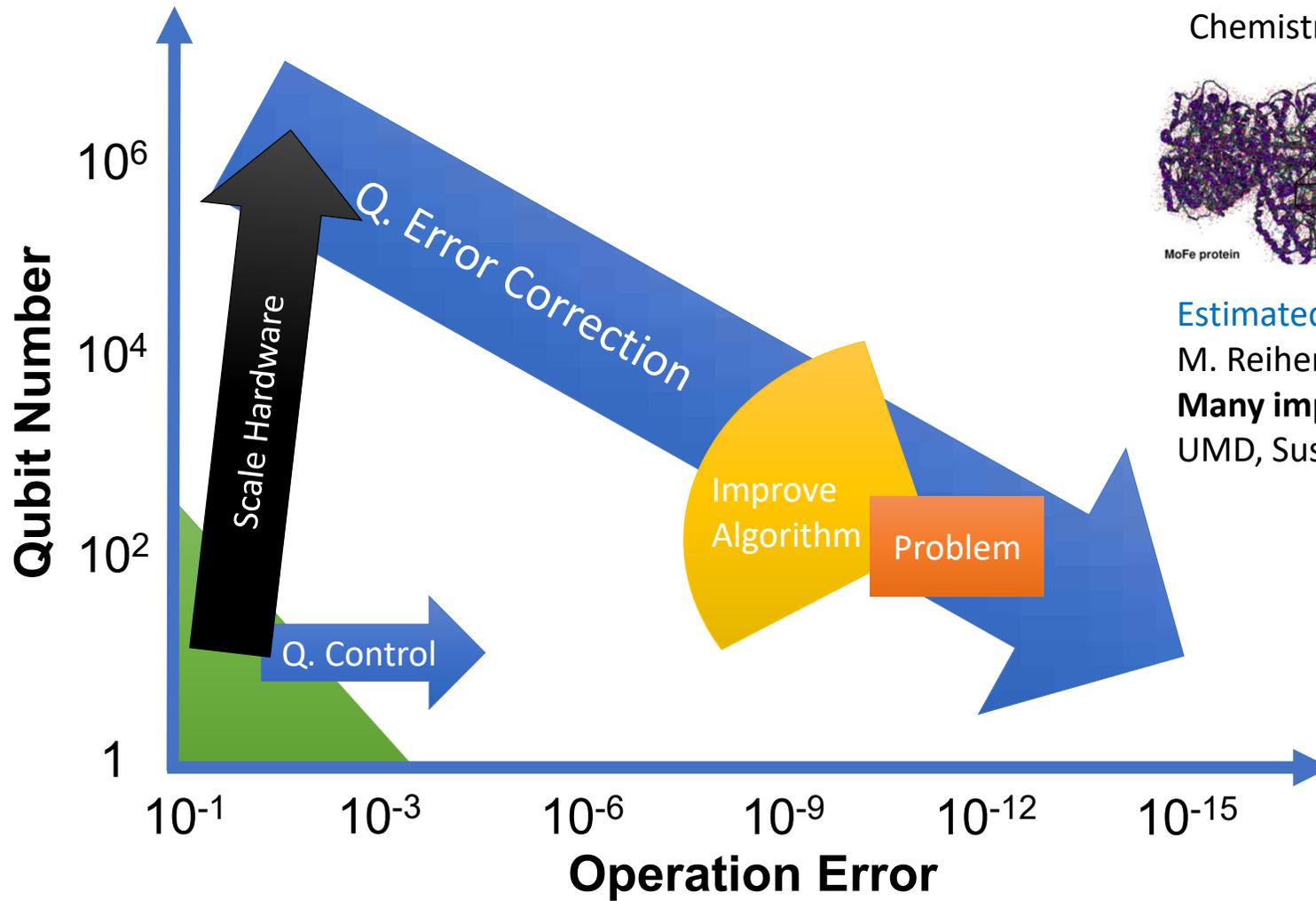
Ken Brown  
Duke University

Disclosure:  
IonQ Advisor

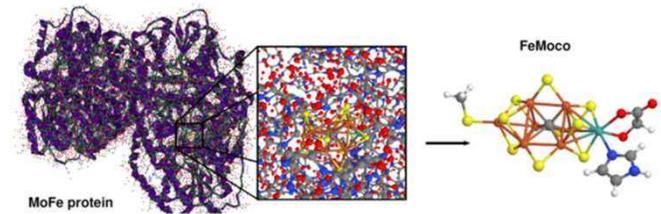
Wanted: More qubits and less error



Wanted: More qubits and less error



Chemistry algorithm



Estimated needed logical gates:  $10^{15}$

M. Reiher et al. PNAS **114**, 7555 (2017)

**Many improvements since 2017**

UMD, Sussex, Macquarie, Google, Microsoft, ..

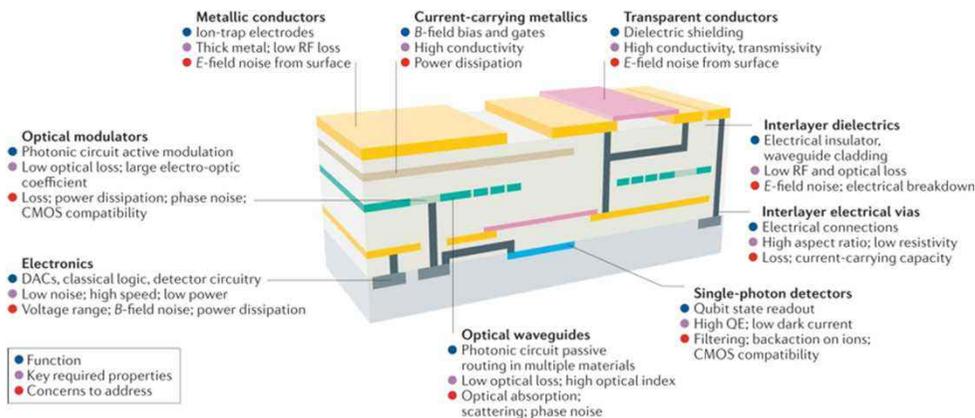


Institute for  
**Robust Quantum  
Simulation**

# Scaling Hardware

## Nature provided (AMO)

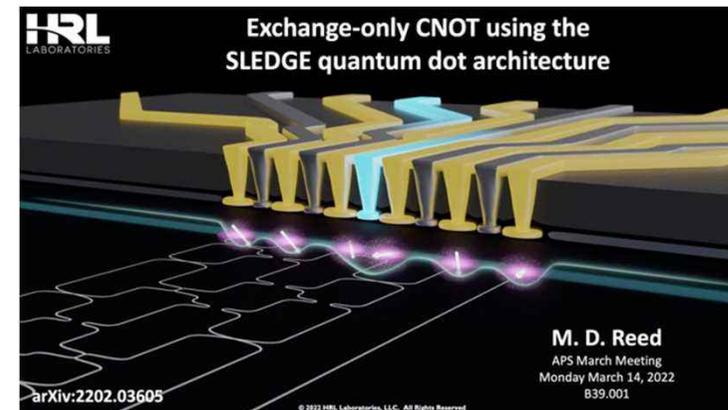
- Every qubit is the same by nature
- Weak coupling to environment
- Challenges in control and confinement of large numbers



K.R. Brown, J. Chiaverini, J. M. Sage, and H. Haefner  
*Nat Rev Mater* **6**, 892–905 (2021).

## Human crafted (CM)

- Every qubit is the same up to manufacturing defects
- Stronger coupling to environment
- Able to “print” as many as can fit on a chip



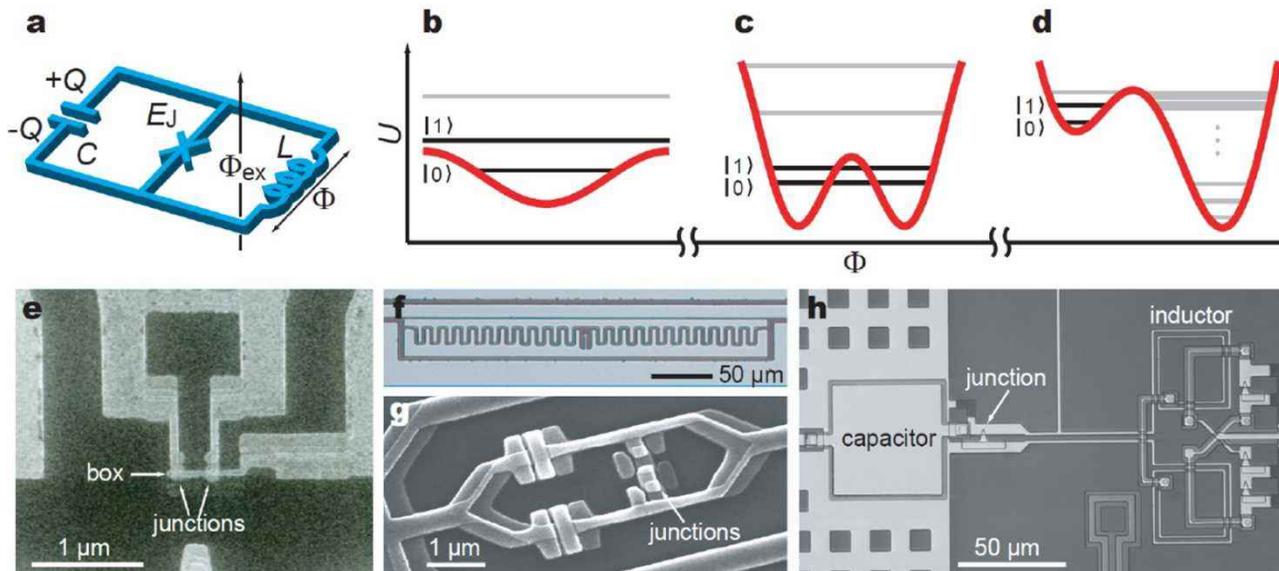
arXiv:2202.03605

# Hilbert space is a vast expanse

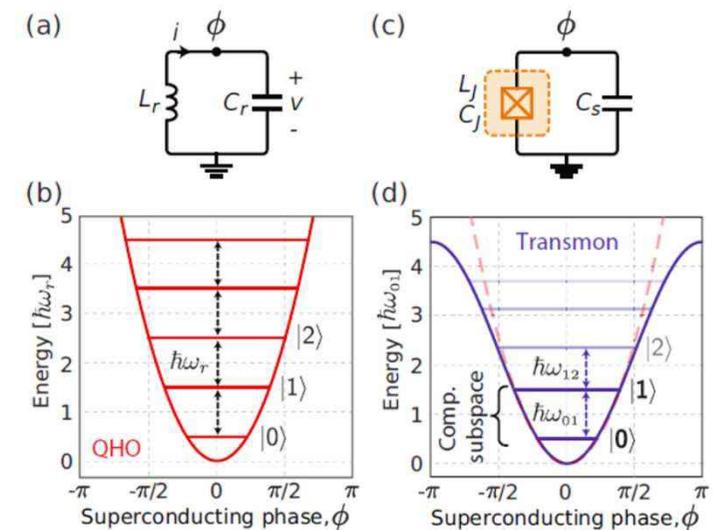


Anselm Kiefer, Dragon, 2001

# Superconducting qubits

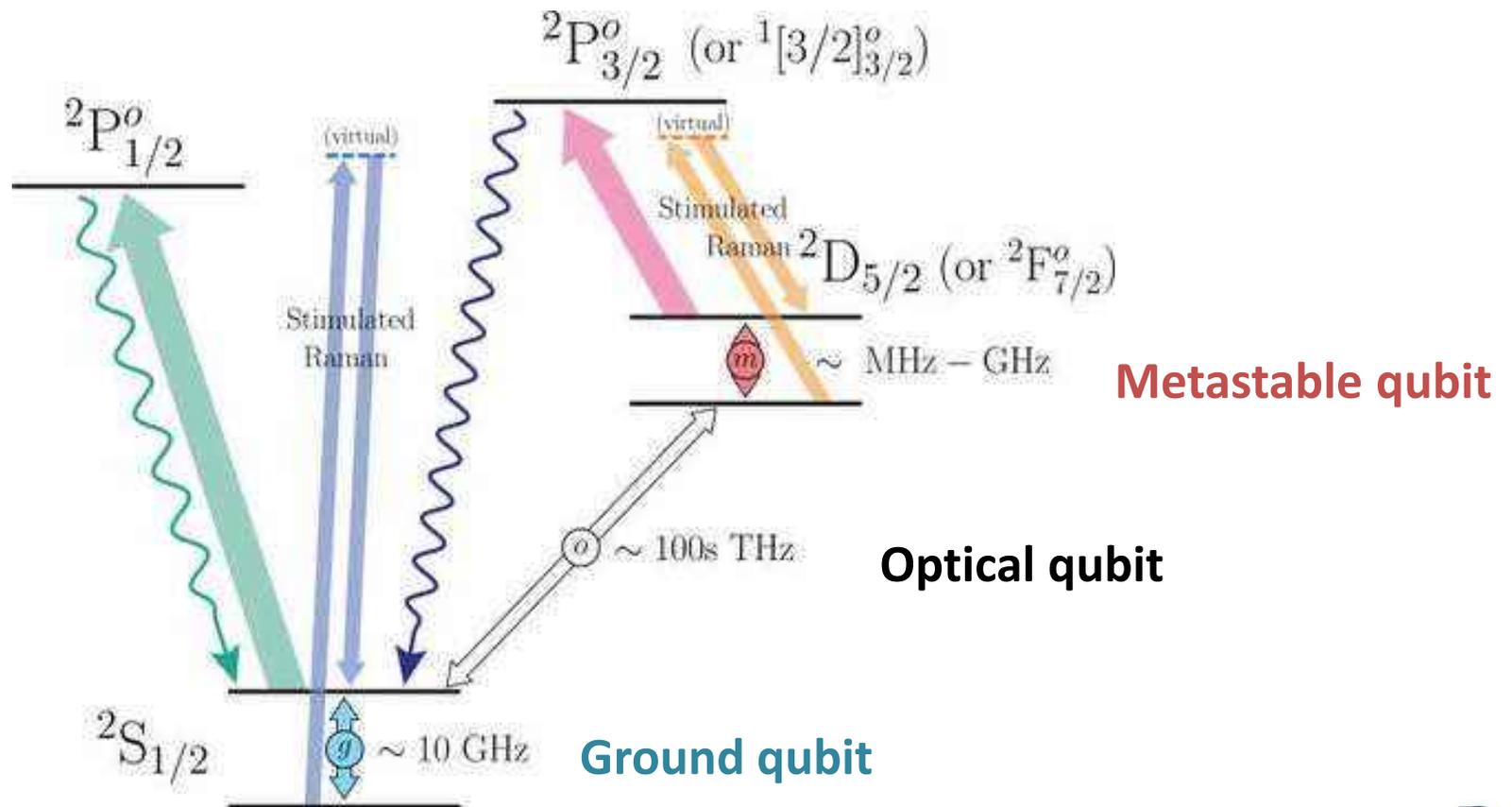


T.D. Ladd et al. *Nature* **464**, 45 (2010)



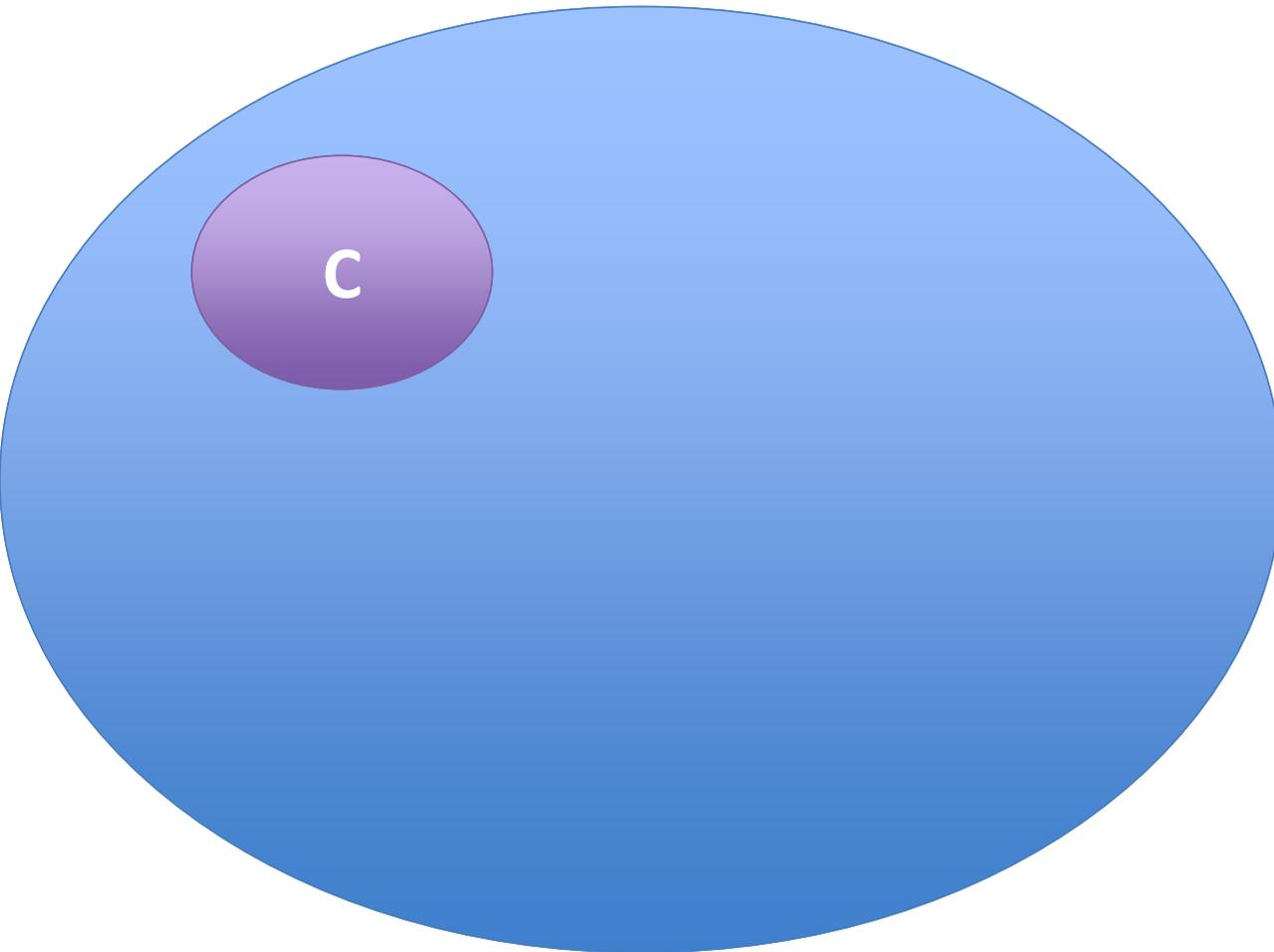
Krantz et al. *Appl. Phys. Rev.* **6**, 021318 (2019)  
MIT Oliver group

# Ion Qubits



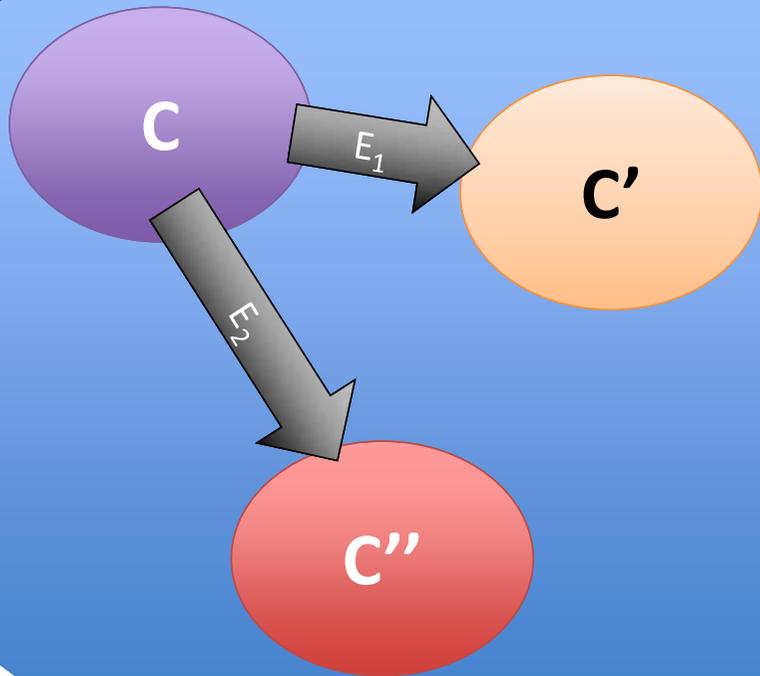
OMG qubits. Allcock et al. Appl. Phys. Lett. **119**, 214002 (2021)

# Quantum Error Correction



A quantum code defines  
a subspace of Hilbert space

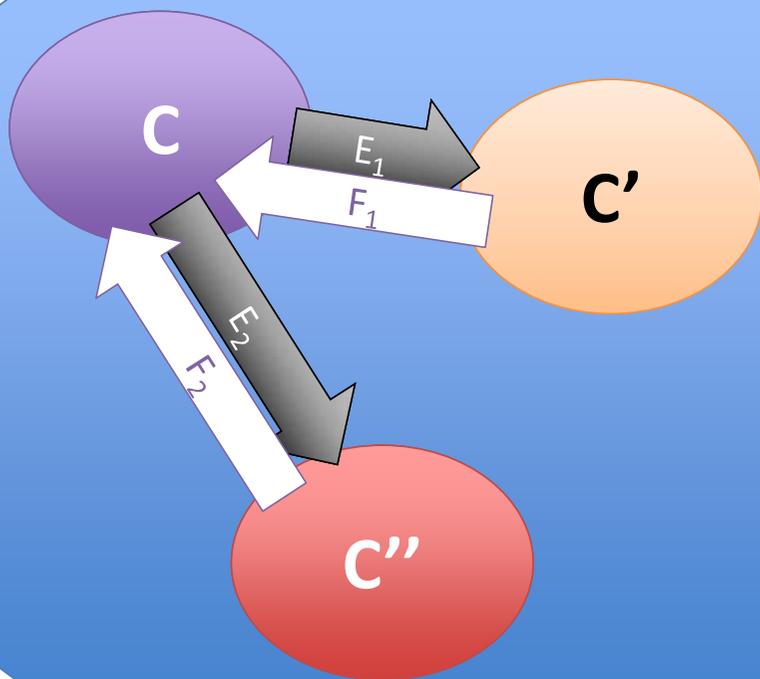
# Quantum Error Correction



A quantum code defines a subspace of Hilbert space

A detectable error maps to an orthogonal subspace

# Quantum Error Correction

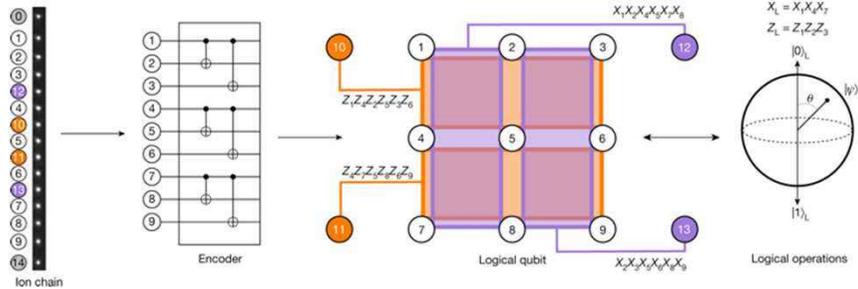


A quantum code defines a subspace of Hilbert space

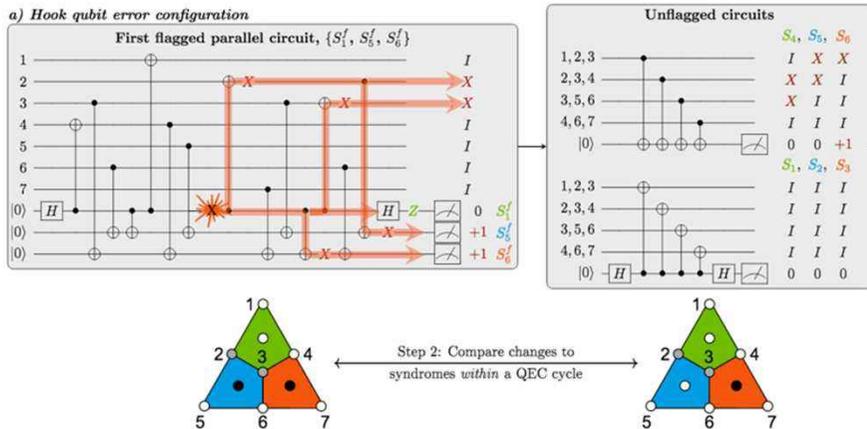
A detectable error maps to an orthogonal subspace

A correctable error can be mapped back

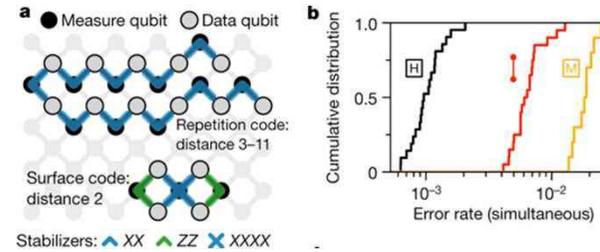
# Quantum Error Correction Now



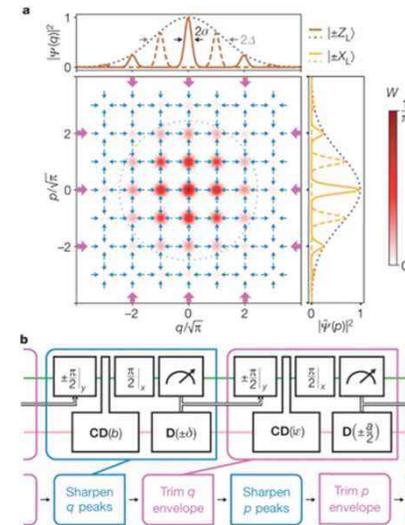
L. Egan et al. Nature (2021)



Honeywell/Quantinuum C. Ryan-Anderson et al. Phys. Rev. X (2021)



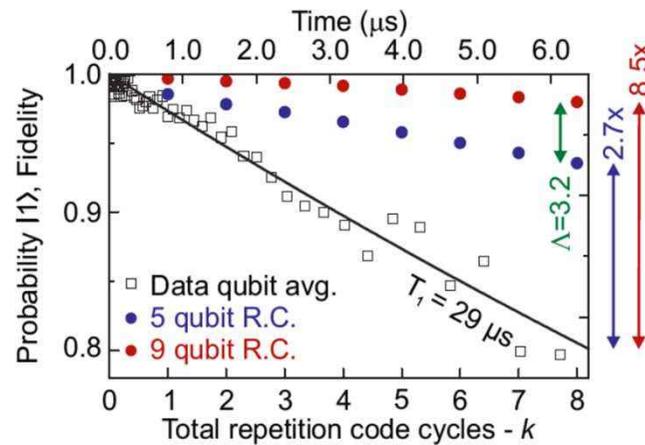
Google Quantum AI Nature (2021)



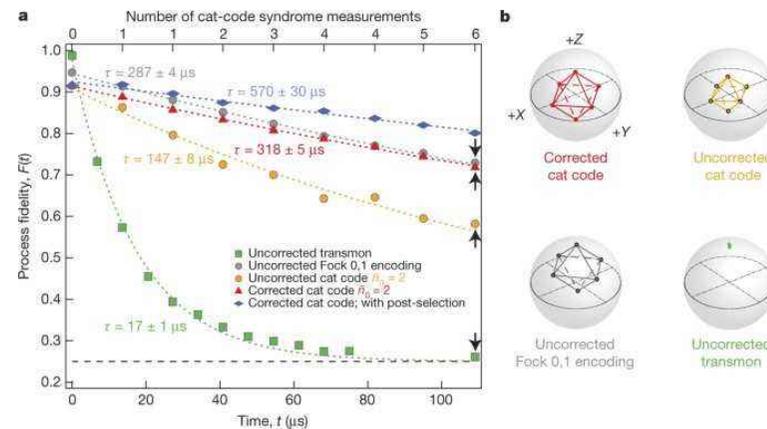
Yale P. Campagne-Ibarcq et al., Nature (2021)

# Quantum Error Correction Tasks

## Improved Memory

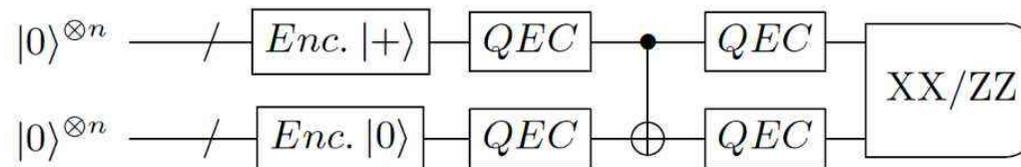


Kelly et al. Nature (2015) (Martinis UCSB)



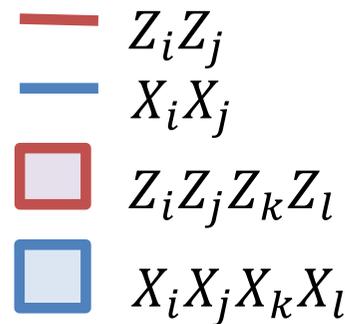
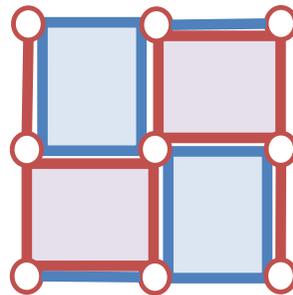
Ofek et al. Nature (2016) (Yale)

## Improved Circuits



# Stabilizer Code

- Stabilizer code  $[[n, k, d]]$ 
  - Codes defined as +1 subspace of  $(n-k)$  commuting Pauli operators
  - $n$  unencoded qubits
  - $k$  encoded qubits
  - $d$  minimal weight of Pauli operator on unencoded qubits that generates a Pauli operator on the logical qubit

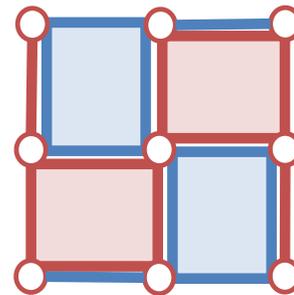


$$Z_L = Z_0 Z_1 Z_2$$

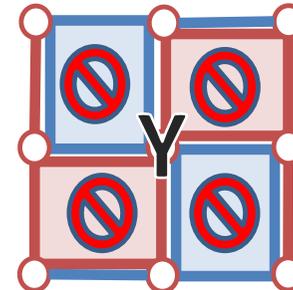
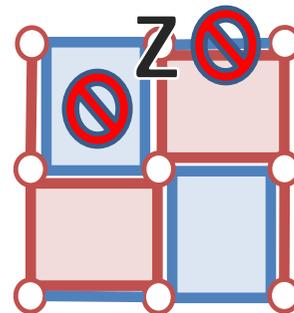
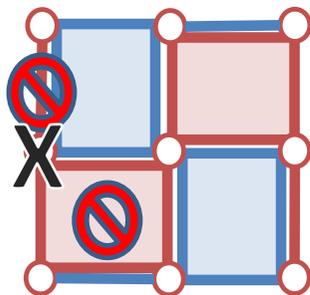
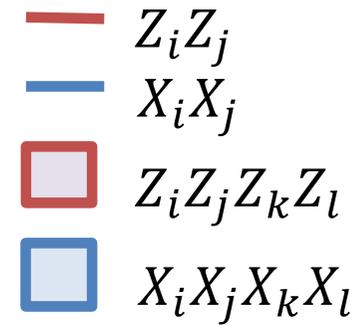
$$X_L = X_0 X_3 X_6$$

Bravyi and Kitaev, arXiv (1998)  
 Bombin and Delgado, PRA (2007)

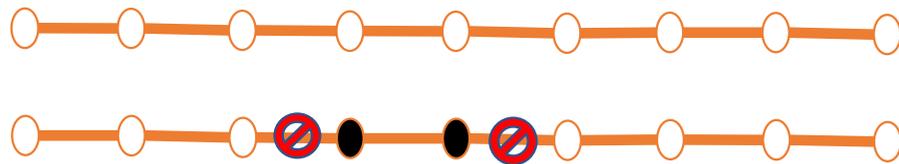
# Surface [9, 1, 3]



Surface



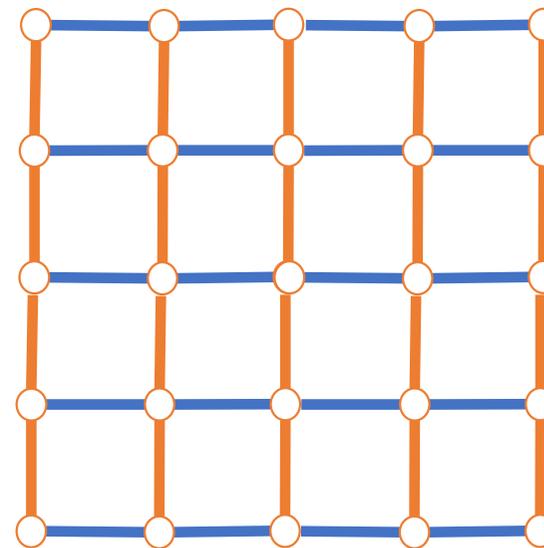
# Codes from spin models



Ising model



repetition code



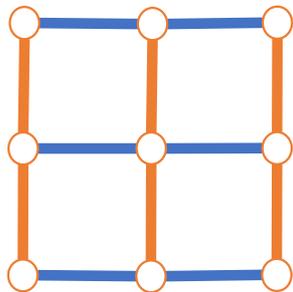
—  $Z_i Z_j$   
—  $X_i X_j$

compass model

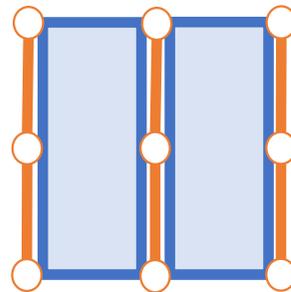


compass codes

# Codes Derivable from Compass Model

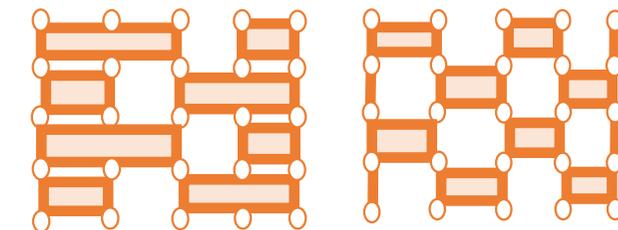
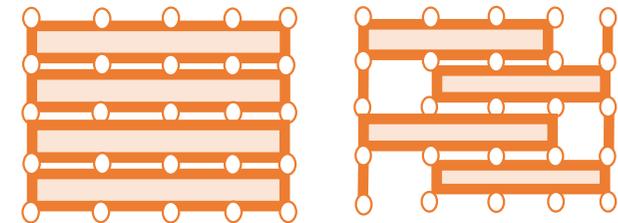


**Compass**



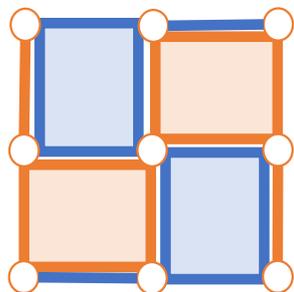
**Shor**

Shor, PRA (1995)



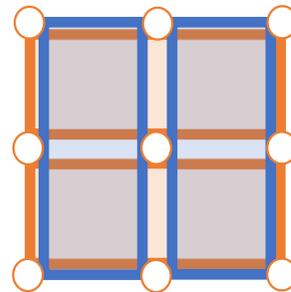
**Many other codes**

Li, Miller, Newman, Wu, KRB PRX (2019)



**Surface**

Bravyi and Kitaev, arXiv (1998)  
Bombin and Delgado, PRA (2007)



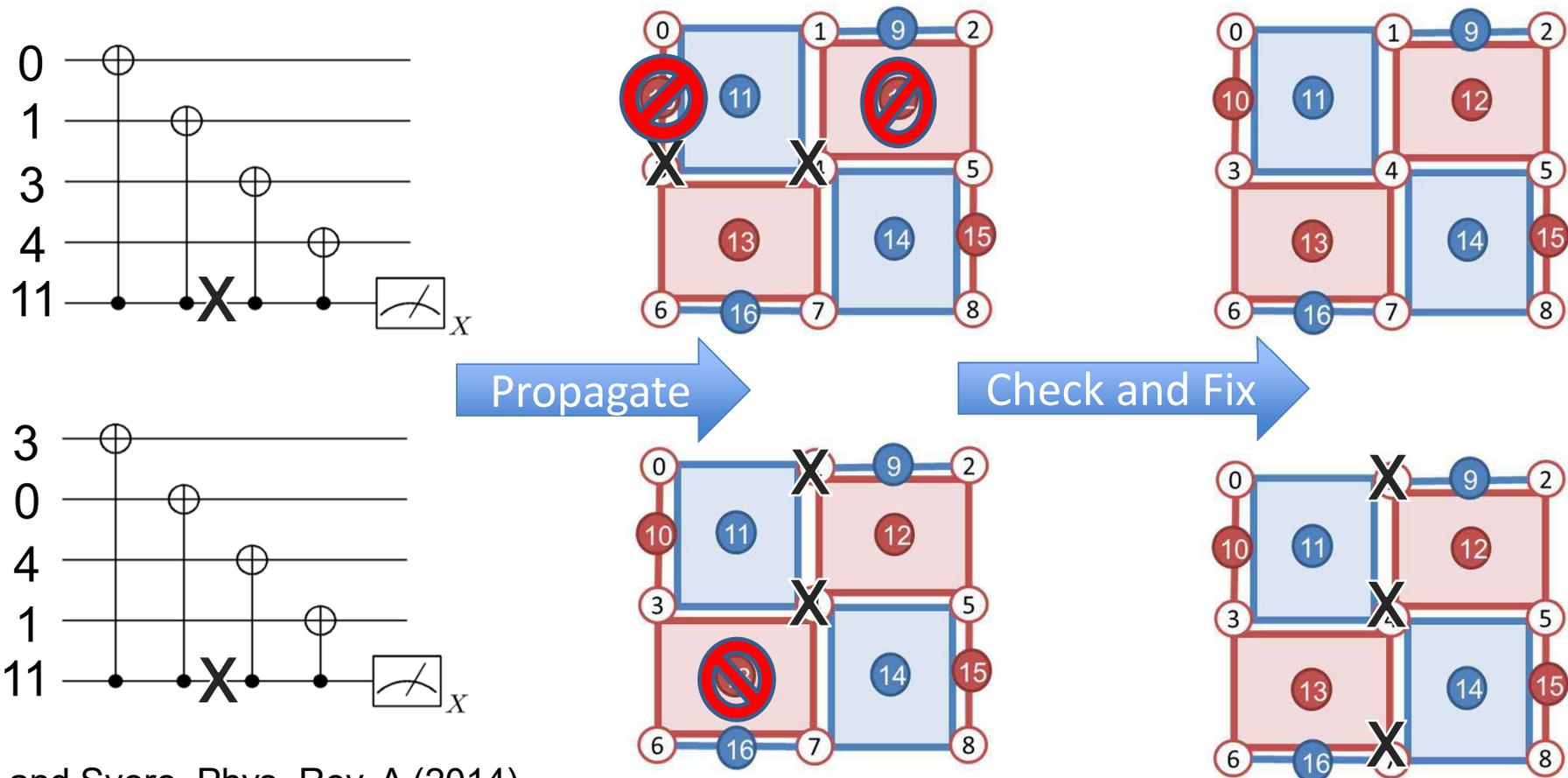
**Bacon-Shor**

Bacon, PRA (2006)  
Aliferis and Cross, PRL (2007)

# Fault-Tolerance

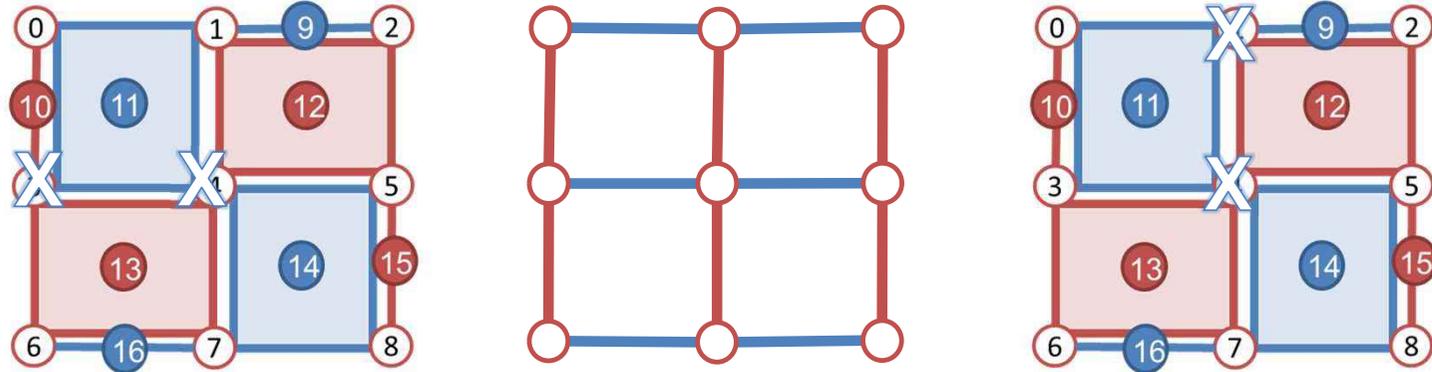
- *Principle for designing processes where the correct answer is guaranteed if there are  $k$  or less faults.*
- Design code, circuit, and decoder for specific fault set (Pauli errors)
- Limits operations (transversal + magic state)
- If error corrected circuit improves performance, you are below a threshold

# Fault-Tolerance and Small Codes



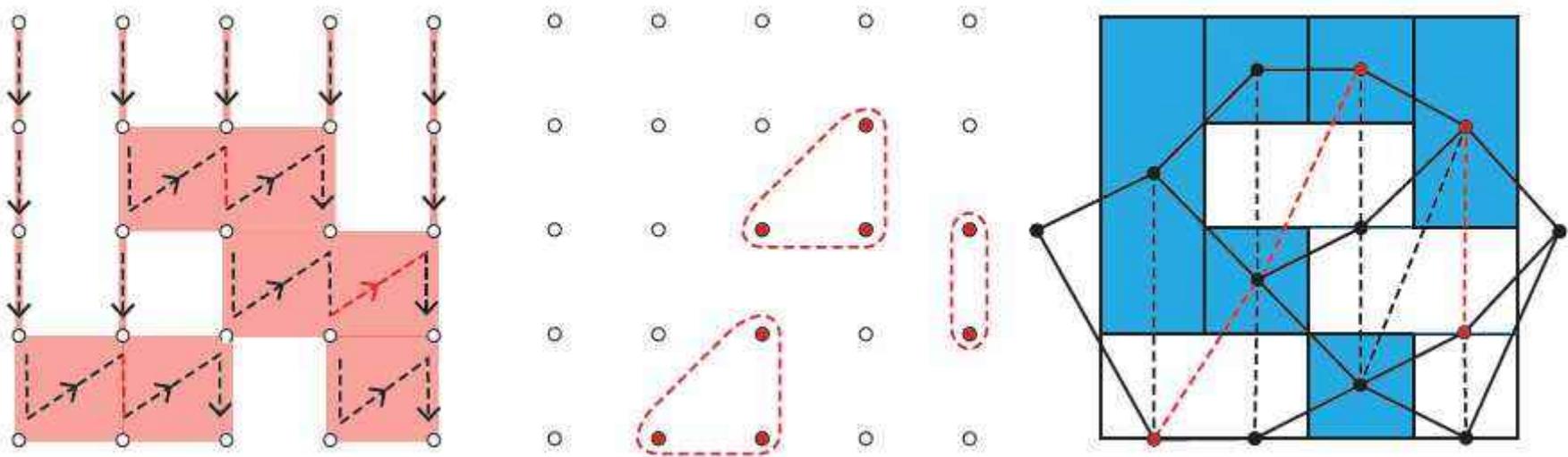
Tomita and Svore, Phys. Rev. A (2014)

# Pattern Determined by Compass Model



Correct order: For X stabilizers, group CNOTs by XX compass model bonds.  
For Z stabilizers, group CNOTs by ZZ compass model bonds.

# Single qubit syndrome checks are FT



Gauges determine the circuit construction.

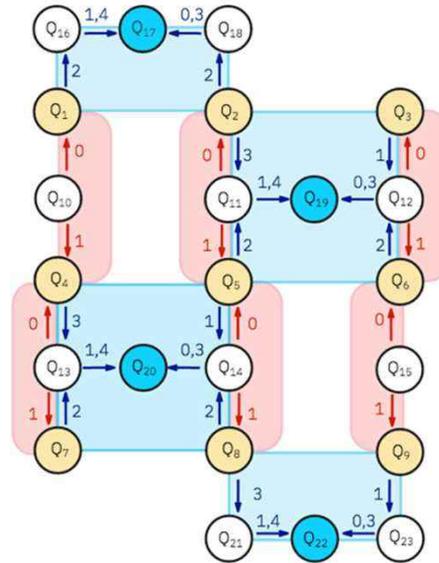
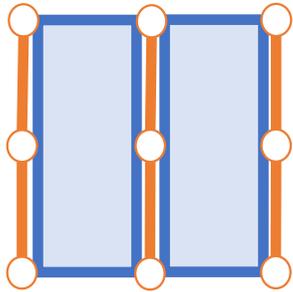
Li, Miller, KRB PRA (2018)

Li, Miller, Newman, Wu, KRB PRX (2019) all compass codes, no algorithm

Huang and KRB PRA (2020) algorithm (+ weighted union-find decoder)

Huang, Newman, and Brown PRA (2020) application to surface code

# Compass Codes Now!



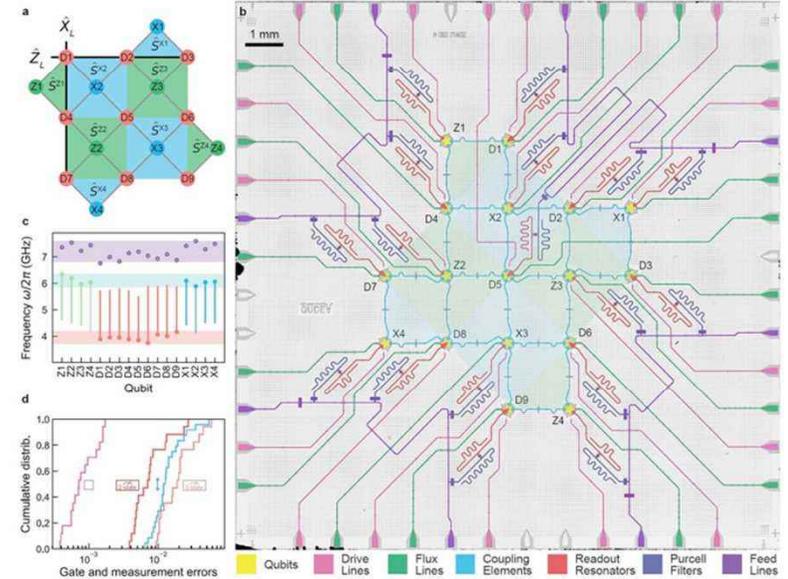
$m$	Prep.	Z Meas.		X Meas.		Majority vote	
		$\mathcal{F}_z$	$\mathcal{F}_z$	$\mathcal{F}_x^+$	$\mathcal{F}_x^-$	$\mathcal{F}_L^+$	$\mathcal{F}_L^-$
3	+	0.951(1)	0.965(1)	0.035(1)	0.9963(4)	0.0037(1)	
	-		0.033(1)	0.967(1)	0.0032(1)	0.9968(4)	
4	+	0.917(2)	0.947(2)	0.053(1)	0.9919(6)	0.0081(1)	
	-		0.051(1)	0.949(2)	0.0076(1)	0.9924(6)	
5	+	0.882(2)	0.936(2)	0.064(1)	0.9976(3)	0.0024(1)	
	-		0.072(1)	0.928(2)	0.0033(1)	0.9967(4)	
6	+	0.806(2)	0.917(2)	0.083(1)	0.9949(5)	0.0051(1)	
	-		0.086(1)	0.914(2)	0.0056(1)	0.9944(5)	
7	+	0.723(2)	0.869(2)	0.131(1)	0.9925(6)	0.0075(1)	
	-		0.132(1)	0.868(2)	0.0076(1)	0.9924(6)	

## [ $m^2, 1, m$ ] Shor code

N Nguyen et al. (Linke group Duke)  
Phys. Rev. Applied 16, 024057 (2021)

## [9,1,3] Heavy Hex

N. Sundaresan et al. (IBM)  
arXiv:2203.07205



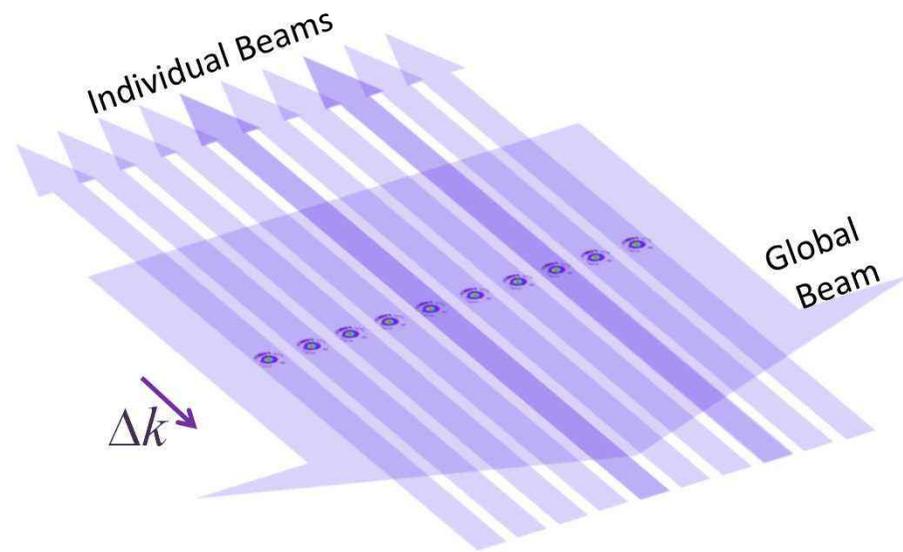
## [9,1,3] Surface Code

S. Krinner, N. Lacroix et al. (Wallraff group ETH)  
Nature 605, 669 (2022)

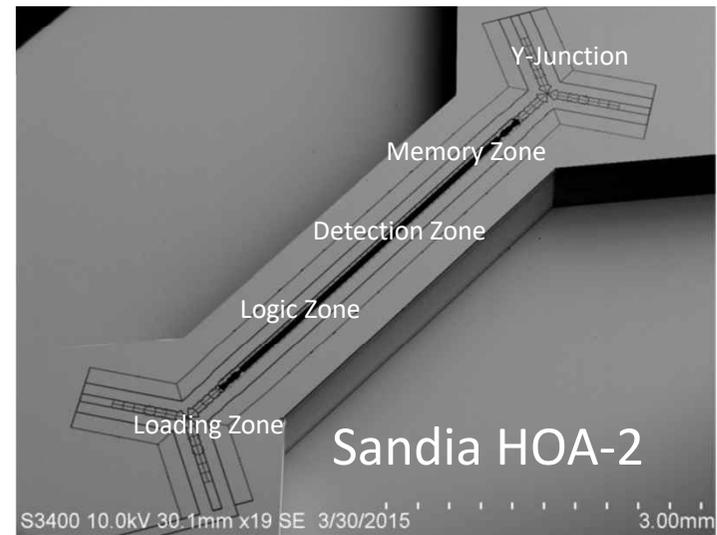
Y. Zhao et al. (USTC), PRL (2022)  
Google [25,1,5] Nature (2023)

[4,1,2] or [4,2,2] UMD, ETH-Zurich, IBM, Sydney, Aachen 2017-2020

# Ion chain quantum computer



Quantum gates: Laser pulses on multiple ions  
amplitude, frequency, phase, pulse shape



Measurement: photon detection  
ion separation by trap electrodes



IARPA EURIQA collaboration: Duke, Harris, ColdQuanta, AOSense

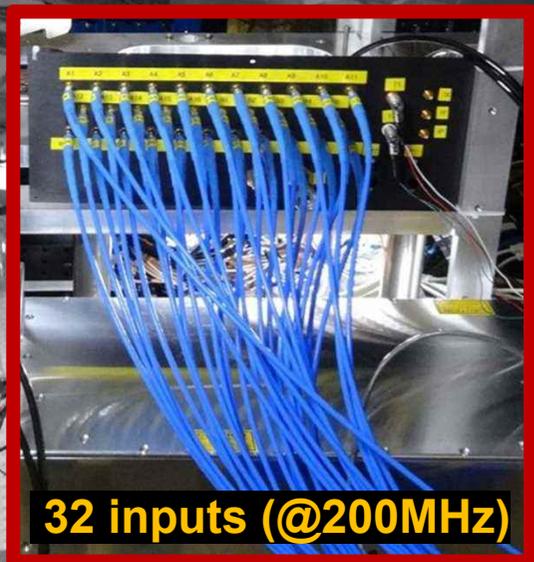


NSF STAQ collaboration: Duke, U Maryland, Tufts, UC Berkeley, Chicago, U New Mexico



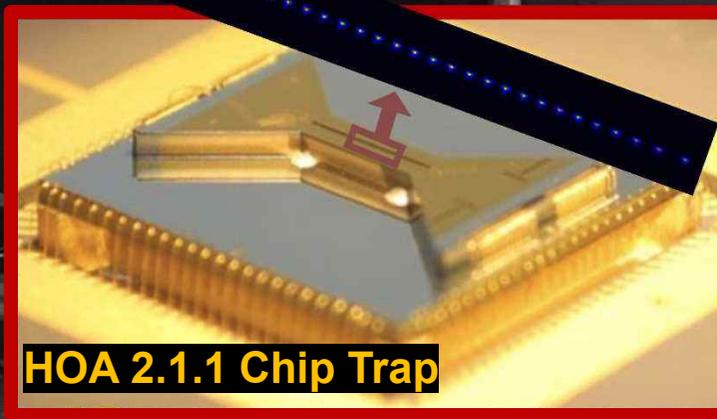
DOE QSCOUT collaboration: Sandia, Duke, Tufts

# Monroe Ion Trap Quantum System



32 inputs (@200MHz)

Chain of 28  $^{171}\text{Yb}^+$



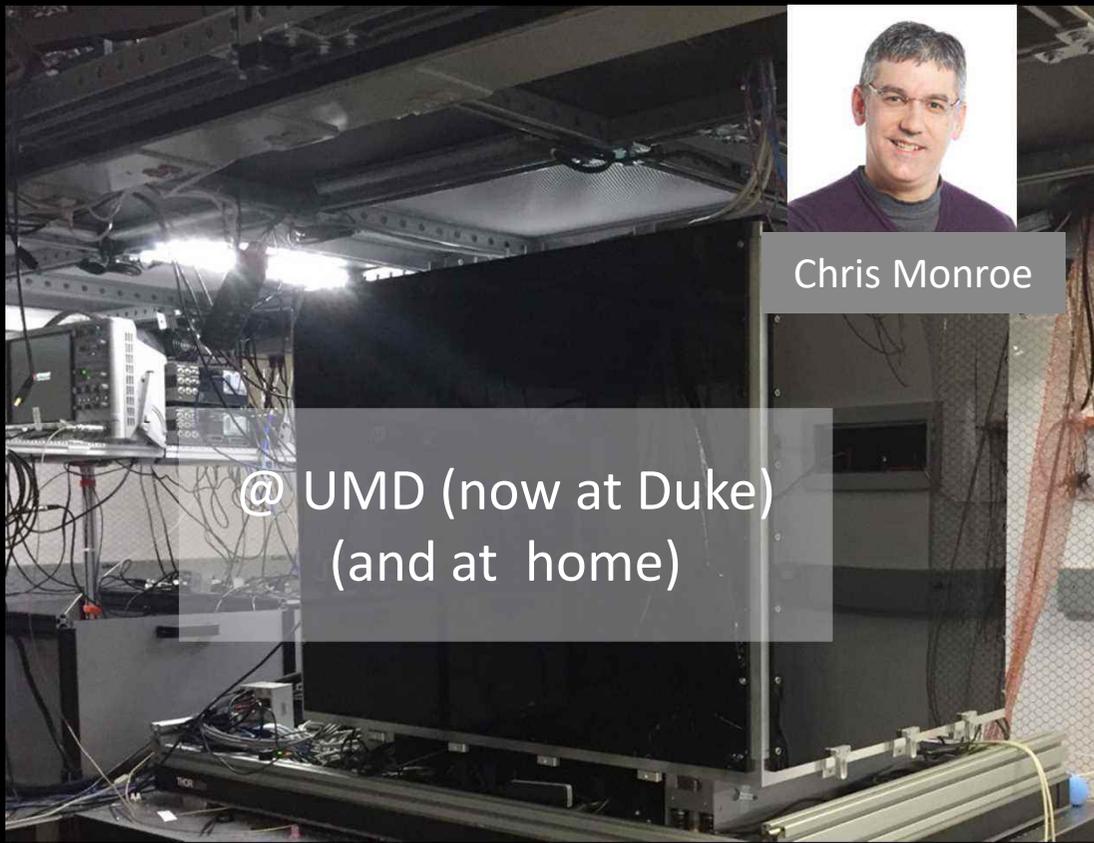
HOA 2.1.1 Chip Trap



32 outputs (PMT counts)



Marko Cetina



@ UMD (now at Duke)  
(and at home)



Chris Monroe



Crystal Noel



Daiwei Zhu



Debopriyo Biswas



Laird Egan

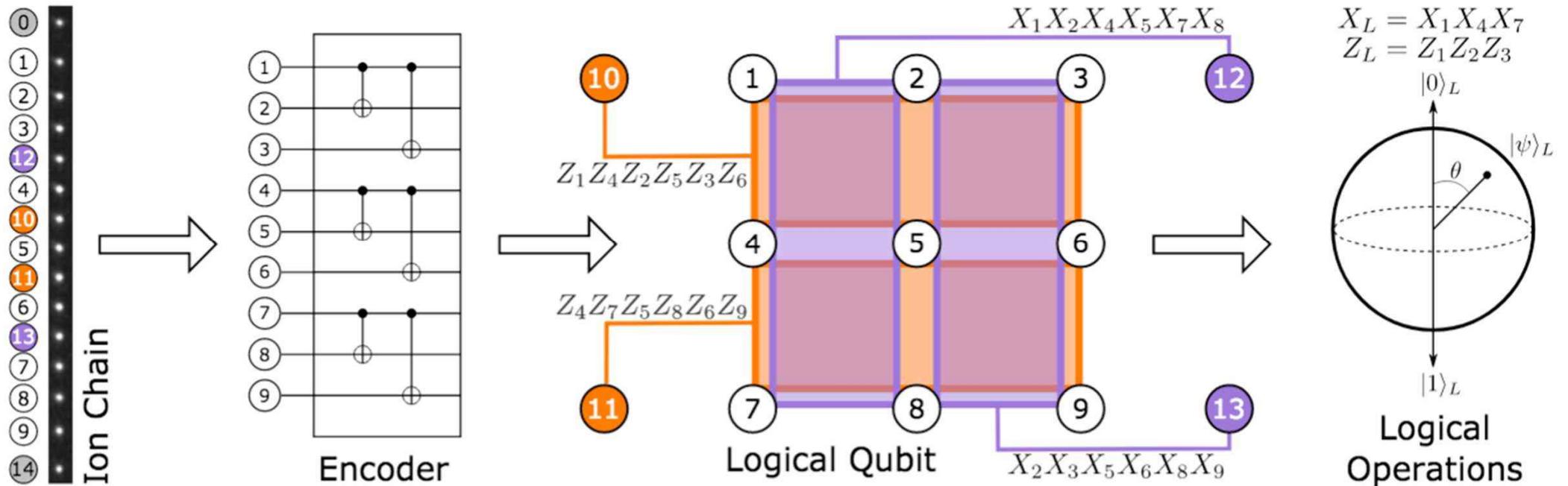


Drew Risinger



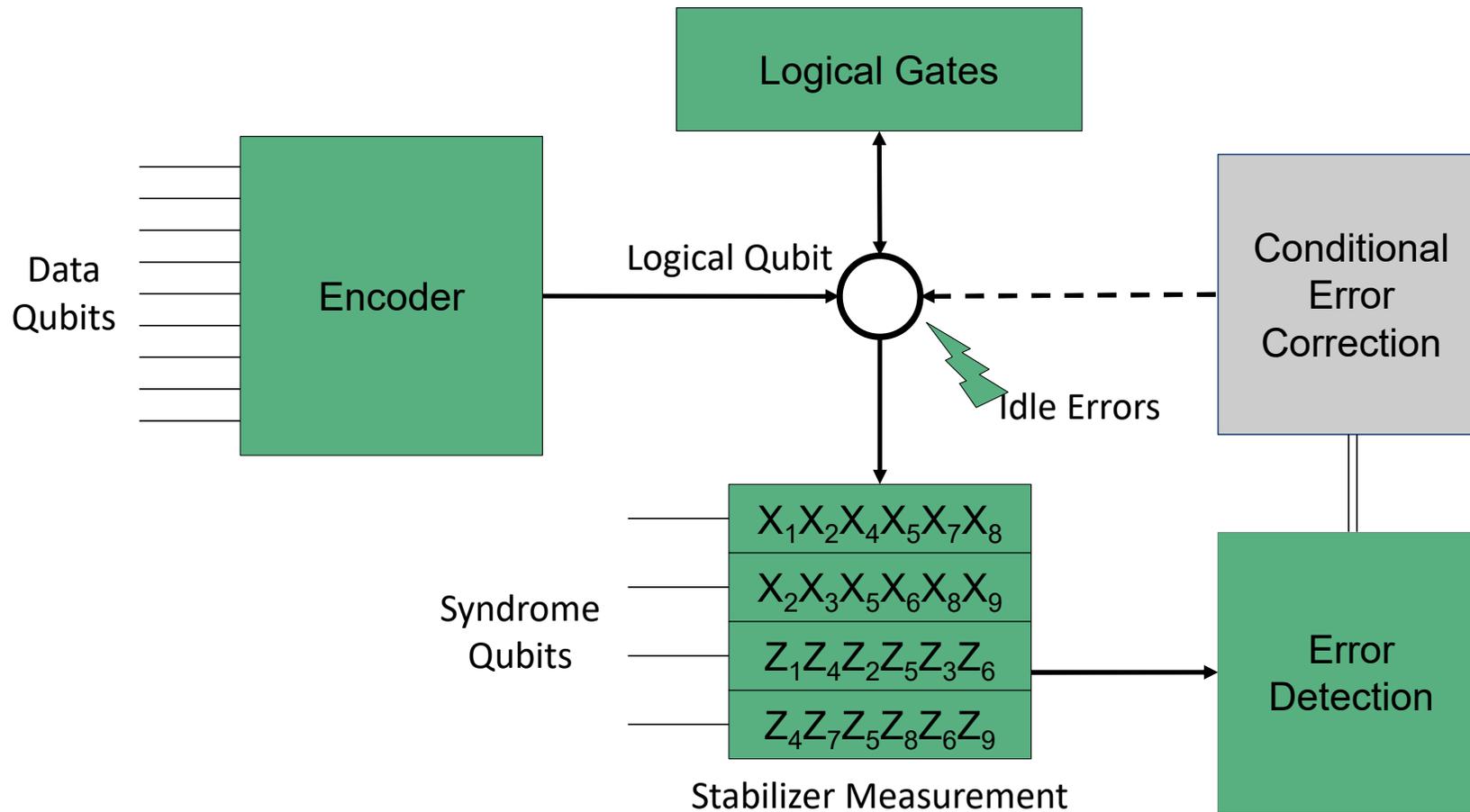
Bahaa Harraz

# Bacon-Shor Experiment

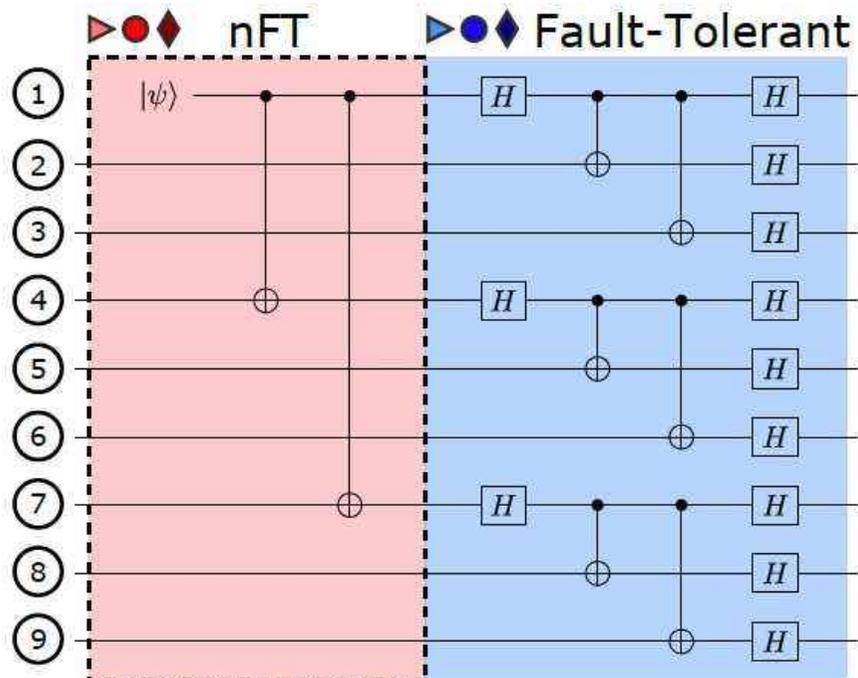


Egan, Debroy, Noel, Risinger, Zhu, Biswas, Newman, Li, Brown, Cetina, Monroe  
 Nature (2021), arXiv:2009.11482. UMD/Duke

# Bacon-Shor Experiment



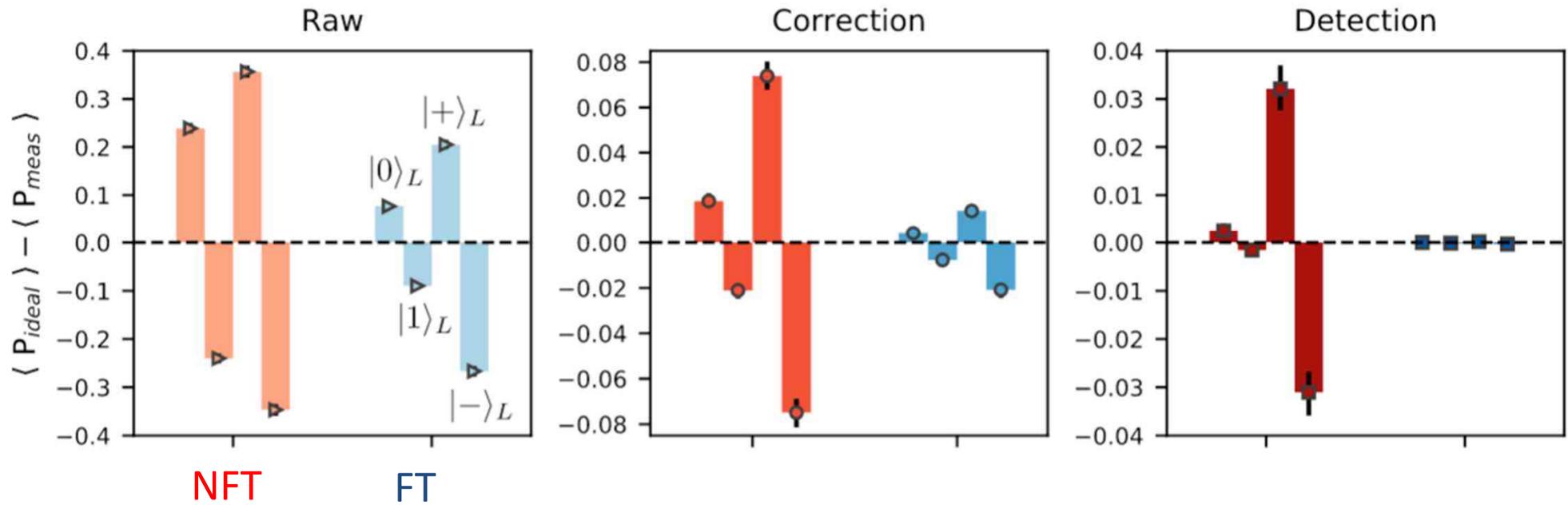
# Preparation & Measurement



Bacon-Shor allows for fault-tolerant preparation of X and Z eigenstates without measurement due to Shor code structure.

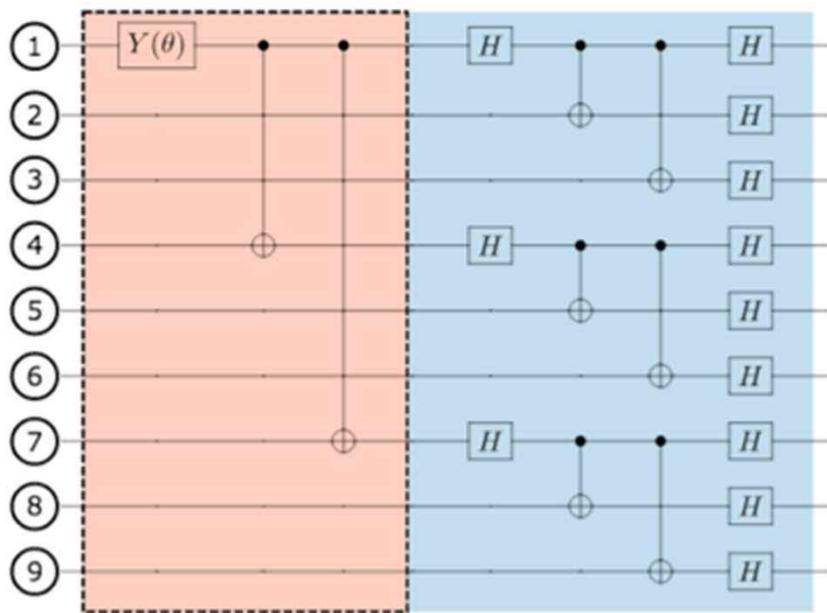
We can generate logical magic states creating a 9 qubit entangled states. The process is not fault-tolerant but necessary for universal computation.

# FT Preparation & Measurement

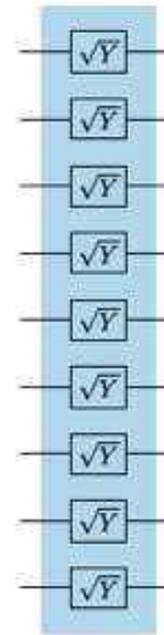


$|+\rangle$  and  $|-\rangle$  prepared by applying FT gate to  $|0\rangle$  and  $|1\rangle$

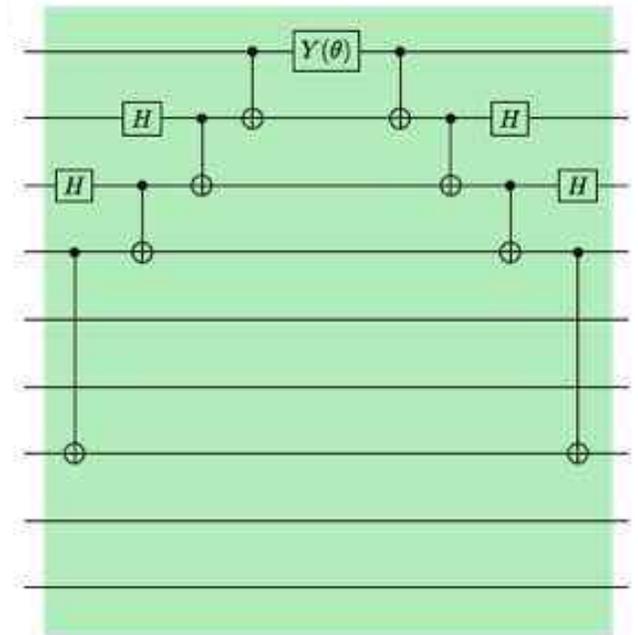
# Preparing other states



direct

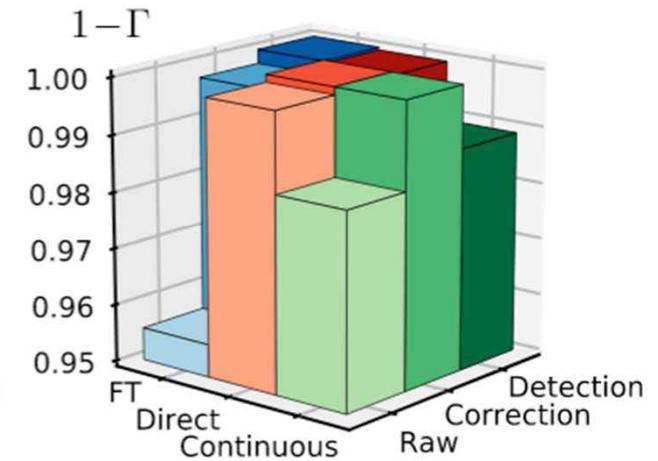
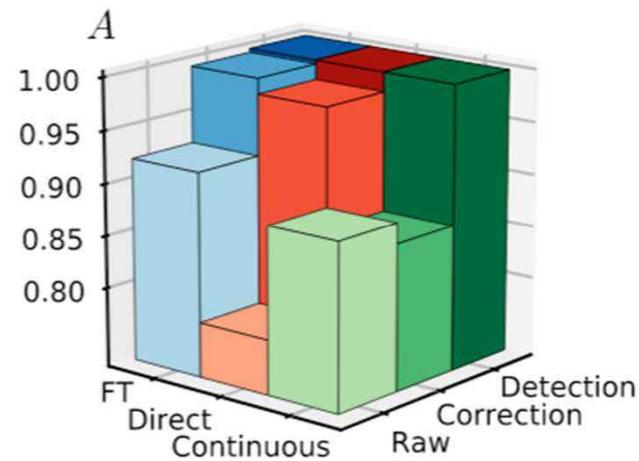
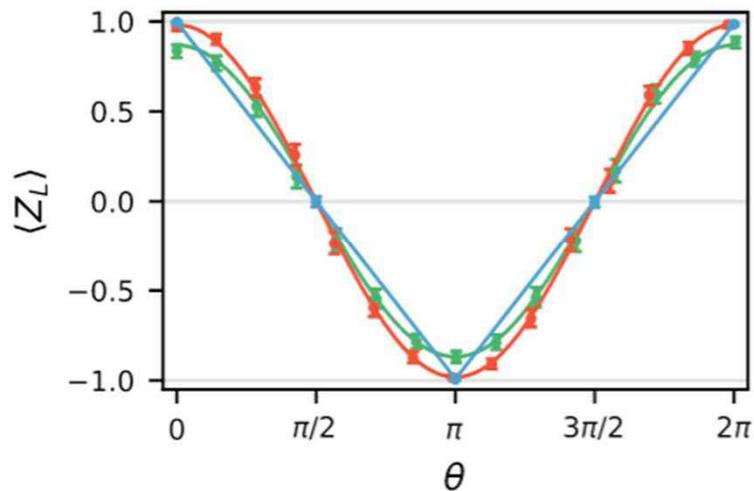


FT



continuous

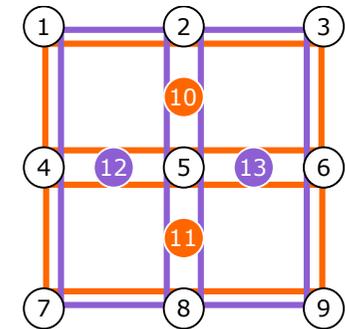
# Comparison of preparations/gates



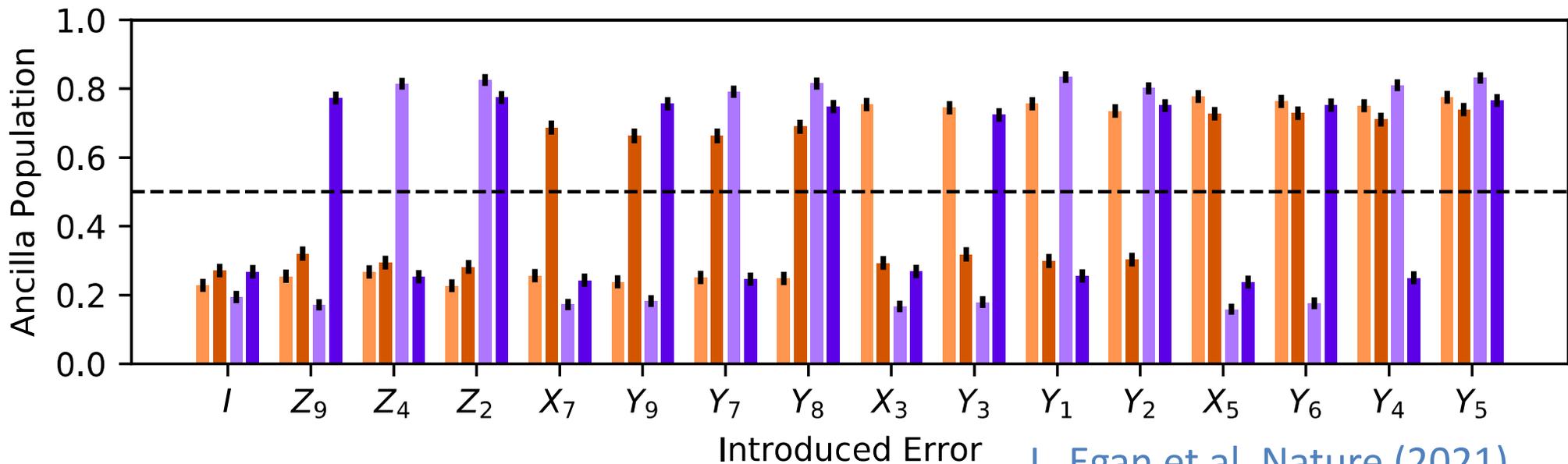
$\pi/2$  fault-tolerant gate error: 0.3%  
 encoded magic state prep error: 2%\*

# Stabilizer checks

Experiment: Encode  $|0\rangle_L$ , introduce an error, map all 4 stabilizers onto syndrome qubits, measure globally

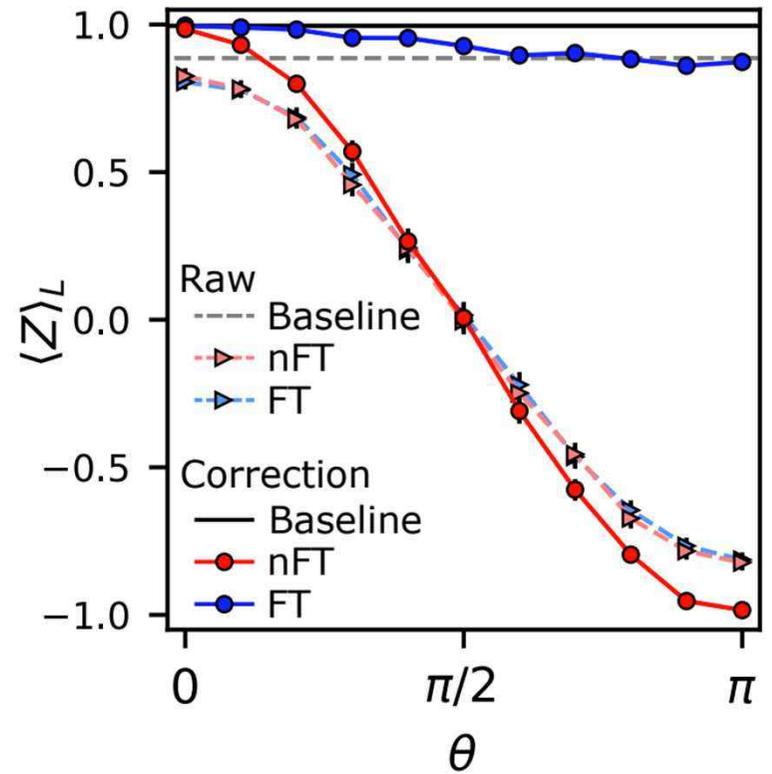
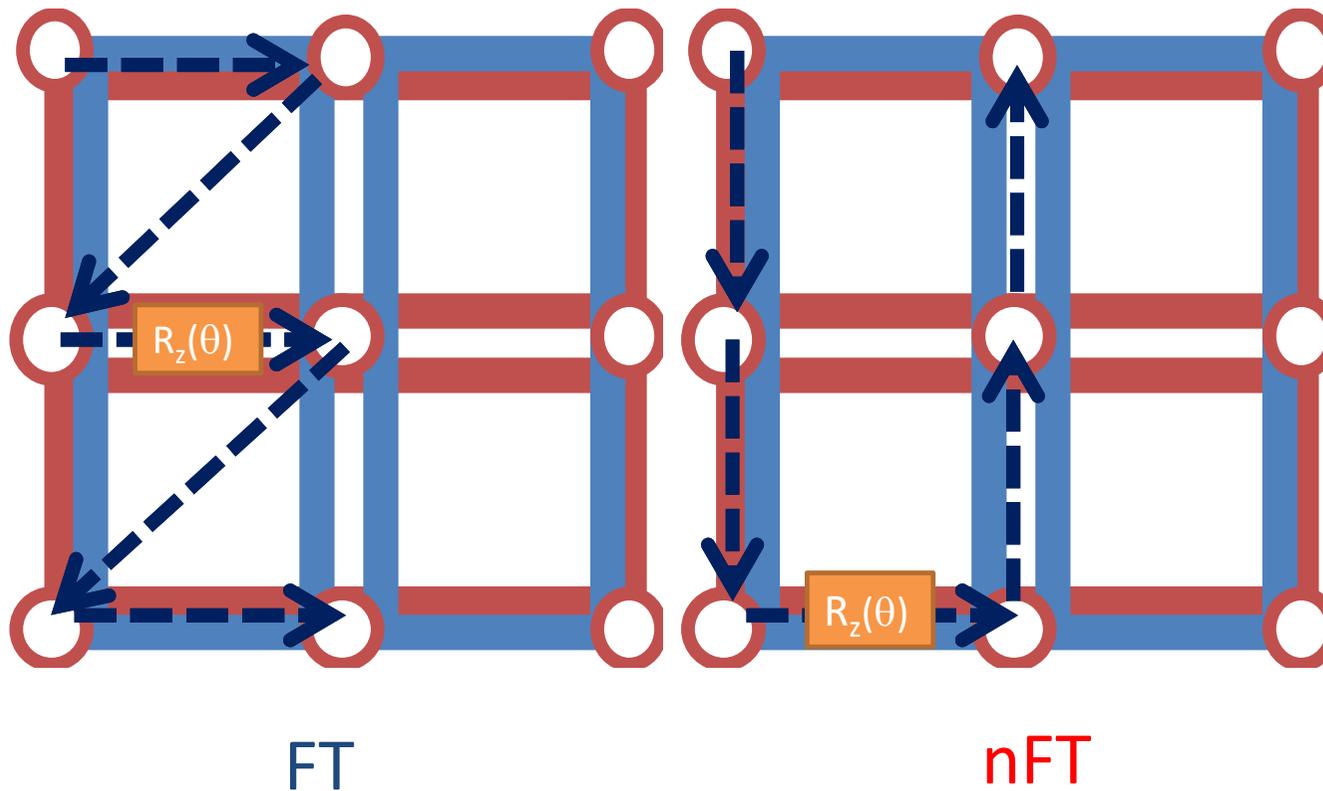


- |  |  |  |  |
|--|--|--|--|
| <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">10</span> | <span style="display: inline-block; width: 15px; height: 10px; background-color: #f4a460; border: 1px solid black; margin-right: 5px;"></span> $Z_1 Z_4 Z_2 Z_5 Z_3 Z_6$ | <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">12</span> | <span style="display: inline-block; width: 15px; height: 10px; background-color: #c8a2ff; border: 1px solid black; margin-right: 5px;"></span> $X_1 X_2 X_4 X_5 X_7 X_8$ |
| <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">11</span> | <span style="display: inline-block; width: 15px; height: 10px; background-color: #d9534f; border: 1px solid black; margin-right: 5px;"></span> $Z_4 Z_7 Z_5 Z_8 Z_6 Z_9$ | <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">13</span> | <span style="display: inline-block; width: 15px; height: 10px; background-color: #6a329f; border: 1px solid black; margin-right: 5px;"></span> $X_2 X_3 X_5 X_6 X_8 X_9$ |

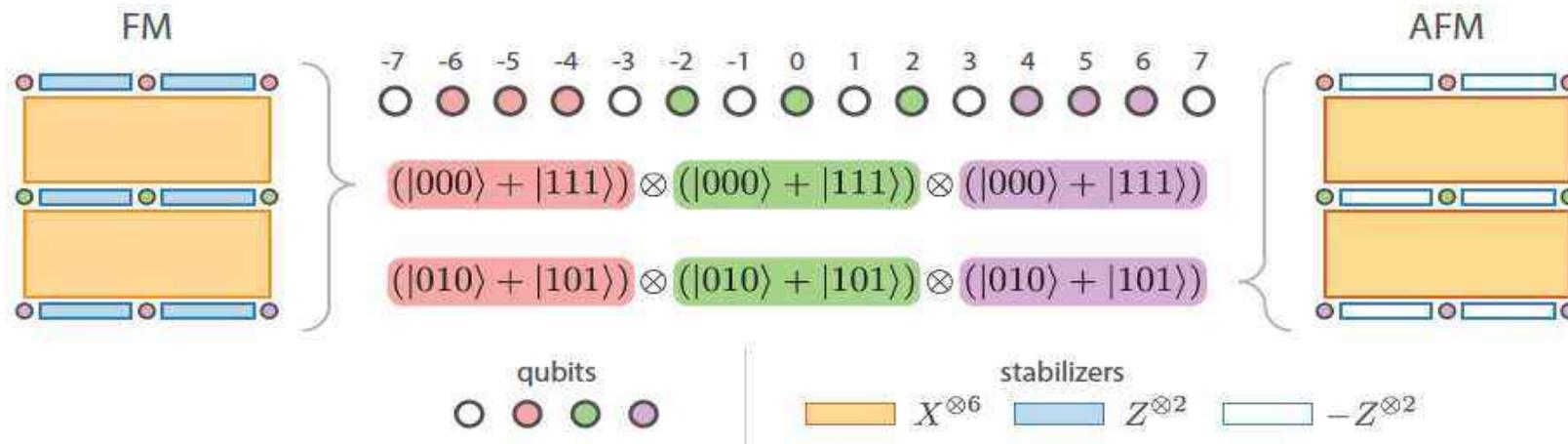


L. Egan et al. Nature (2021)

# Fault-tolerant gadgets



# Reducing Correlated Errors



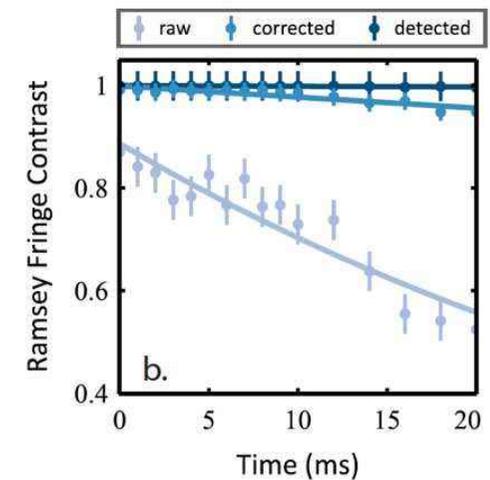
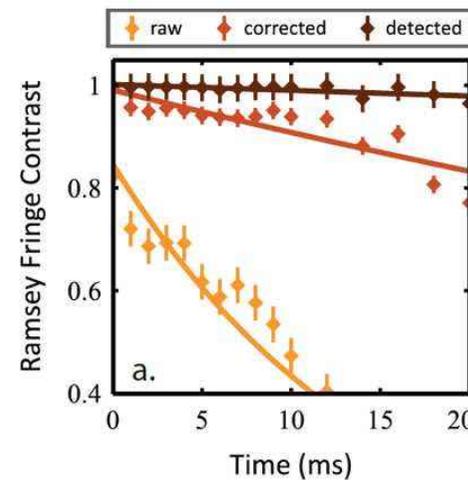
Increase of encoded corrected lifetime by a factor of 4.

Comparable to our single qubit life time.

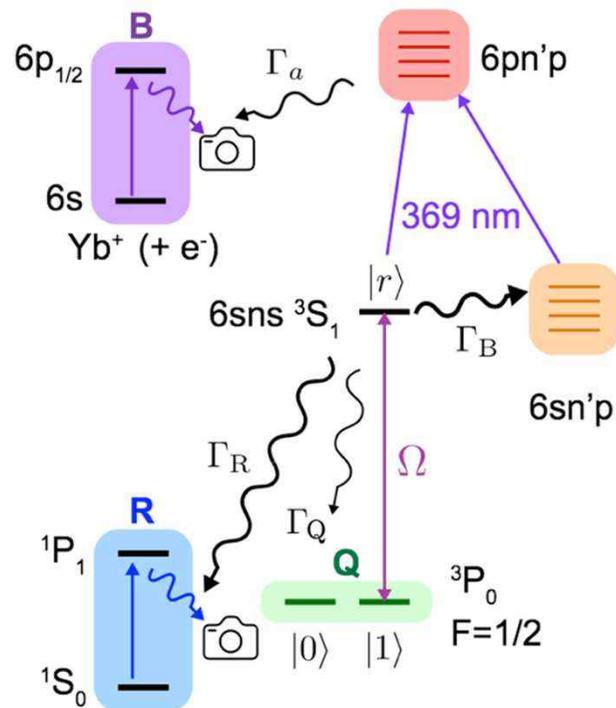
In standard independent Pauli error model these codes should have the exact same behavior.

[D. Debroy et al. Phys. Rev. Lett. \(2021\) arXiv:2105.05068](#)

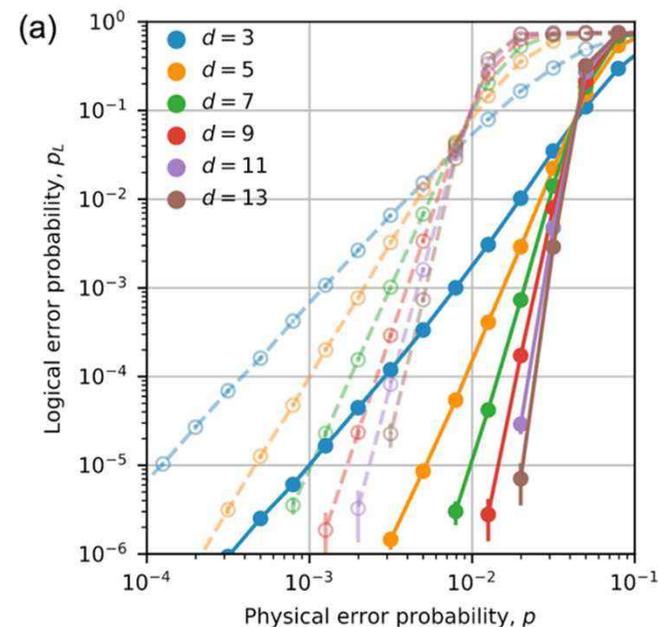
See also Hu, Liang, Rengaswamy, Calderbank, arXiv:2011.00197



# Matching codes and decoders to physical errors



Wu, Kolkowitz, Puri, Thompson  
arXiv:2201.03540

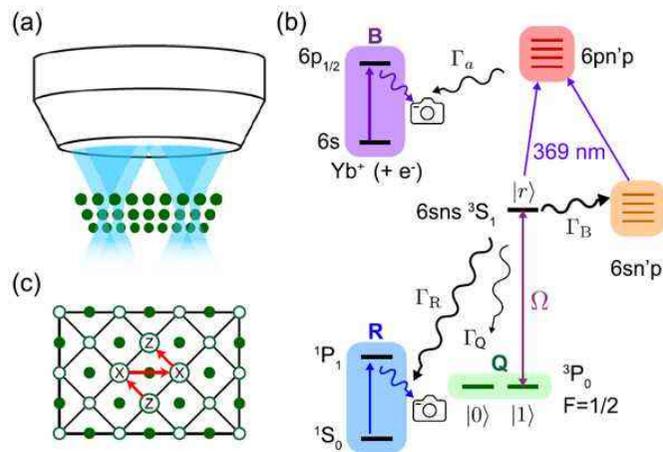


Conversion to erasure greatly  
improves threshold

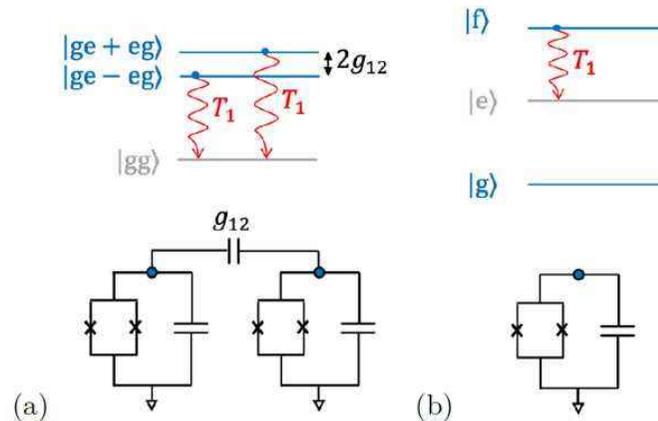
Other ideas: XZZX codes, bias preserving cat codes, designer cluster states

# Erasure Conversion Trilogy

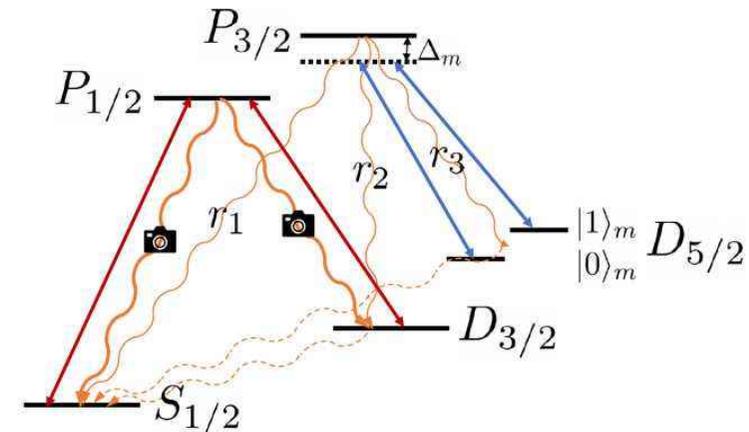
## Rydberg atoms



## Superconducting qubits



## Trapped ions



Y. Wu et al., Nat. Comm. 13, 4657 (2022)

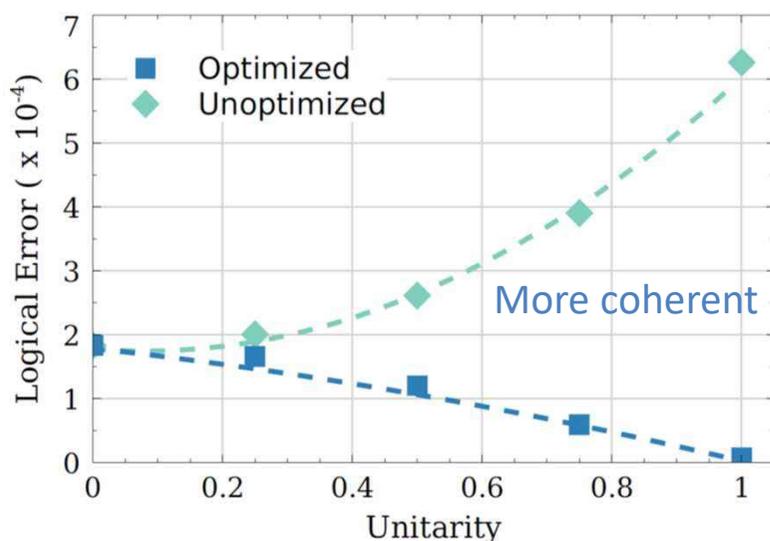
A. Kubica et al., arXiv:2208.05461

M. Kang, W.C. Campbell, KRB, arXiv:2210.15024

- Design qubits s.t. physical noise causes **leakage** outside the qubit in a **detectable** way
- Additional operations turn leaked state to  $\hat{I}/2$  in qubit subspace  $\rightarrow$  **erasure!**

# QEC Beyond Pauli Errors

## Coherent Errors



Not a problem for larger codes

[Beale et al., PRL 2018](#)

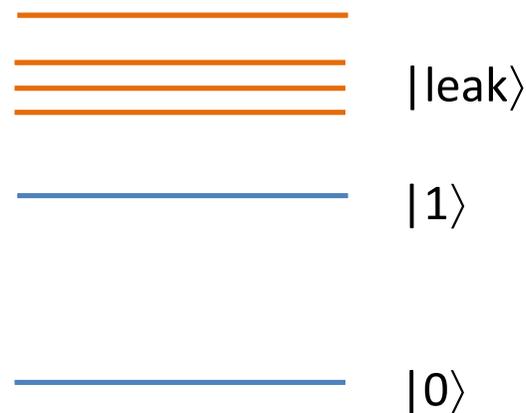
Not a problem if you try to fix it

[Debroy, Li, Newman, Brown PRL 2019](#)

Minimize with codespace

[Debroy, Egan, et al. PRL 2021](#)

## Leakage Errors



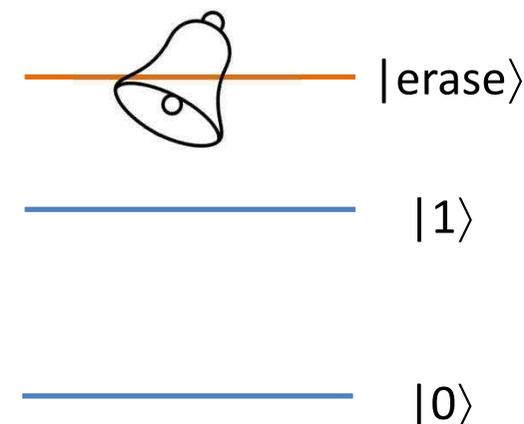
Leakage vs memory errors

[Brown and KRB, PRA 2018, PRA 2019](#)

Subsystem codes for leakage

[Brown, Newman, KRB NJP 2019](#)

## Erasure Errors



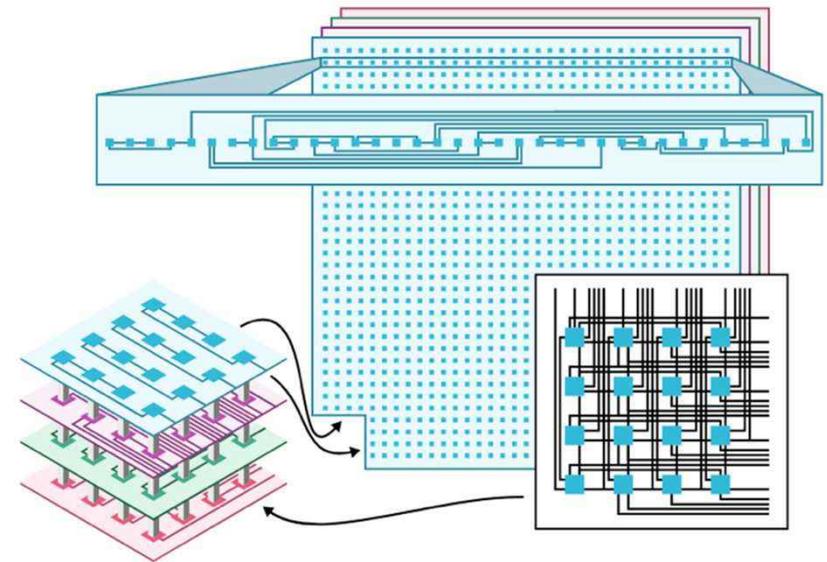
Erasure errors

[Kang, Campbell and KRB](#)

[arXiv:2210.15024](#)

# QEC Now!

- Experiments have reached a point where QEC is possible
- Designing hardware that enables finite rate codes remains challenging
- QEC procedures tailored to physical errors can reduce the overhead
- Fault-tolerant gate design for finite rate codes remains an open question
- Theory of fault tolerance is also transforming



Tremblay, Delfosse, Beverland  
arXiv:2109.14609

**1000 qubits/logical qubit to  
50 qubits/logical qubit**

**Low overhead constructive codes**

Panteleev and Kalachev, arXiv:2111.03654

Leverrier and Zemor, arXiv:2202.13641

Yamasaki and Koashi, arXiv:2207.08826

and more..... dynamic research area

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@kenbrownquantum

EPIQC

STAQ

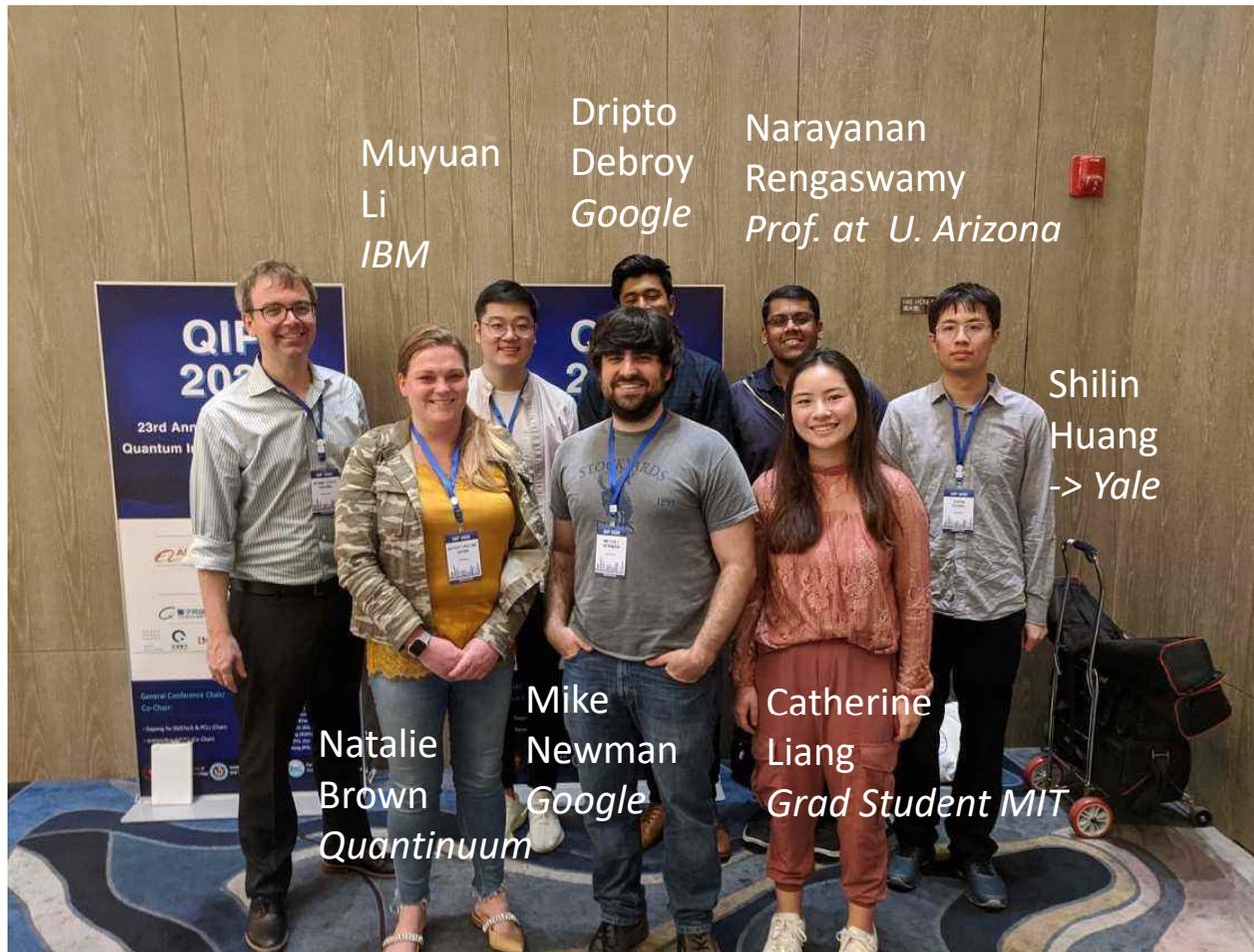
Institute for  
Robust Quantum  
Simulation



Mingyu Kang



Wes Campbell  
UCLA



Muyuan  
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