Goal8 Realization of a society safe from the threat of extreme winds and rains by controlling and modifying the weather by 2050.

Actuator Location Optimization for Large Degree-of-Freedom Fields

R&D item



Here begins our new MIRAI

1. Development of Actuator Location Optimization Algorithm and Its Application to Modeling and Weather Problems

Progress until FY2023

1. Outline of the project

Since a weather field has a large number of degrees of freedom, weather simulations take quite a long time. Therefore, it is not practical to use an algorithm that performs a huge number of weather simulations for actuator selection. We will develop actuator-position-selection algorithms that can be applied to such large systems based on a mathematically formulated problem in this theme. To efficiently evaluate the performance of the algorithm, simulation models of relatively small systems are also developed in parallel with the development of the actuatorselection algorithm. For these test models, the developed algorithm shows that the actuation at the selected location can significantly modify the field with a probability of over 99% better than that at a randomly selected location (Figure 1). We finally show the effectivity of the developed algorithm for a weather model.



2. Outcome so far

We have developed some numerical algorithms to optimize actuator placement in a linear model by considering the dual problem of sensor placement optimization, and then have evaluated their efficiency with some test models. First, we proposed an algorithm for time-invariant linear model with impulse forcing. In this algorithm the determinant of a matrix associated with right singular vectors is maximized by greedy method. This algorithm was applied to a linearized Ginzburg-Landau model and it was shown that the system can produce an output with various modes. Further, it was indicated that the algorithm can be applied to a large-scale system such as compressible axisymmetric jets when randomized singular value decomposition is used.

Then, we developed an optimization algorithm for a timevariant linear system, which may be considered as a perturbation system of a nonlinear system such as meteorological phenomena. In this algorithm actuators are placed so that the reach set of the time- series output is maximized. We applied this algorithm to Lorenz-96 model, which is known as a simple model of meteorological





phenomena, and numerically showed that the model can be controlled efficiently.



Figure 3: Right singular vector in an axisymmetric jet



Figure 4: Response in Lorenz-96 model

3. Future plans

In collaboration with the themes "Mathematical problem formulation for actuator location optimization" and "Development of an evaluation method for optimized actuator location based on weather simulation" in this project, we will apply our actuator-location optimization algorithms to weather fields and demonstrate that the quantity of state can be varied more efficiently when actuators are placed by the proposed method than when placed randomly.



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2. Mathematical Problem Formulation for Actuator Location Optimization

Progress until FY2023

1. Outline of the project

Actuator-position optimization problems are formulated to find actuator positions that efficiently change states such as the temperature and the velocity of the atmosphere. By clarifying the evaluation metrics, it becomes possible to quantitatively evaluate the effect of the actuator placement and to develop an algorithm to determine the actuator placement. If the formulated mathematical problem is too complex, the problem cannot be solved. Hence, we formulate the problems that are simple enough to solve, in coordination with the other themes.

2. Outcome so far

(1) Analysis of actuator-position optimization problems for linear time-invariant systems

Objective functions in the actuator-position optimization problems for linear time-invariant systems were scrutinized by using relationship between objective functions of the actuator-position optimization problems and their dual sensor problems. When any impulse and consecutiveinterval inputs is employed and any terminal and consecutive-interval outputs is employed, a matrix representing input-output relationship allows us to evaluate the effect of actuator position. In special cases these matrices coincide with the well-known Gramians in control theory. For example, if we consider a consecutive-interval control input of which norm is limited, the size of the reachable set of states corresponds to the size of the controllability Gramian (Figure 1, top). Furthermore, we investigated metrics to evaluate the magnitude of these matrices and clarified their relationship to existing metrics used in sensor-position optimization and the magnitude relationship between the metrics (Figure 1 bottom).

- Reachable set of linear time-invariant (LTI) system
 - LTI system: $x[k+1] = Ax[k] + Bu[k], \ x[0] = 0$
 - Reachable set under constraint: $\mathcal{R} = \left\{ \xi \in \mathbb{R}^n \mid \xi^\top \underline{W}^{-1} \xi \leq 1 \right\}$



Figure 1: Reachable set and metrics for linear time-invariant system

(2) Formulation of actuator-position optimization problems for nonlinear systems

To account for nonlinearity of weather dynamics, an actuator-position optimization method using the ensemble singular vector method was investigated. This method uses a nonlinear model to make predictions for multiple particles and calculates modes with the greatest input-output sensitivities, allowing for dimensionality reduction. Furthermore, this method has an advantage of not requiring an adjoint model. However, we found that the lack of the use of adjoint models results in a problem: right singular vectors, which are important for determining the actuator position, are not calculated accurately. In addition, we formulated an actuator position optimization problem based on tangent linear models. The tangent model describes the variation when a small control input is applied to a nonlinear system. Utilizing the knowledge obtained for linear time-invariant systems in (1), we formulated actuator-position optimization problems in which reachable sets in tangent models, which are linear time-variant, are maximized (Figure 2). Although these problems require the singular value decomposition using adjoint models, it has low computational complexity.



Figure 2: Outline of problem based on tangent model

3. Future plans

Given that our project will finish in one year, our project as a whole plans to proceed with an actuator-position optimization problem based on a tangent model in which an adjoint model is used, rather than a problem based on a nonlinear model, which was found to have a challenge in the calculation accuracy of the right singular vectors. On the other hand, since problems based on nonlinear models are important when a weather field changes significantly due to control, we will continue to study problems based on tangent models in this theme for the future applications.



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R&D item



3. Construction of Evaluation Method for Optimized Actuator Placement on Weather Simulator

Progress until FY2023

1. Outline of the project

We will construct an evaluation technique of the effectiveness of input at optimized actuation locations on a realistic weather simulator. The weather fields to be modified and controlled in this project are those of extreme weather such as local heavy precipitation. Reproduction of such kind of meteorological phenomena is not easy in simulations at current. Therefore, it is necessary to reliably reproduce extreme weather for evaluating the effects of inputs on extreme weather fields on a weather simulator at first. In addition, since there are various weather models and each model has advantages and disadvantages, the selection of the weather model is also important. In this theme, we will clarify various parameters such as the weather model to be used, the boundary conditions, the size of the computational domain, the resolution, etc. Then, the optimization method of actuator locations developed in other themes of our project will be applied to verify the effectiveness of the proposed method.

2. Outcome so far

We have selected a weather simulator and conducted trial simulations. We are mainly using the WRF model, which was developed mainly by the National Center for Atmospheric Research (NCAR) in the United States. This is because the WRF model provides the adjoint model required for typical methods of calculating sensitivity to inputs, and the optimization method of actuator locations developed in this project also anticipates the use of the adjoint model.

Figure 1 shows an example of a sensitivity analysis performed using WRFPLUS software for the local heavy precipitation that occurred in western Japan in July 2018. The computational region was limited to a small area to reduce computational costs. Simulations were performed for six hours from noon on July 5th, which was used as the base trajectory to calculate the sensitivity regarding the change in the water vapor on the ground to the accumulated rainfall. Random disturbances were applied to the amount of water vapor on the ground. The time integration was performed using a tangent linear model, and the accumulated precipitation at the terminal state was calculated as the initial state for the time integration using the adjoint model.





The figure of the terminal state of the tangent linear model shows that the precipitation on the Hiroshima and Okayama area changes significantly due to the disturbance in the amount of water vapor on the ground, while the figure of the terminal state of the adjoint model shows that this change occurs due to a change in the water vapor amount on the southwest side.

We also proceeded with the construction of a tool to perform weather simulations by applying arbitrary input to the initial state of a nonlinear simulation using the standard WRF model. At present, we have confirmed that it is possible to intervene in the water vapor amount and evaluate its influence in the simulation. We are currently working on supporting inputs in limited locations which is intended for the sparse actuator. We are also getting ready for the evaluation of the actuator optimization method scheduled for the final year. We will apply the developed optimization method of actuator locations in a weather simulator and verify its effectiveness.

3. Future plans

In the future, we will optimize the actuator location for extreme weather fields reproduced on a simulator in several different scenarios in which local heavy precipitation occurs. In the simulations, we will demonstrate that by combining the sensitivity analysis tool using the WRF model developed in this theme with the randomized singular vector method developed in other research themes in this project, it is possible to efficiently calculate the mode of the sensitivity distribution for inputs even in large-degree-of-freedom fields such as meteorological fields. By combining with an actuator position optimization algorithm developed in this project, suitable locations for actuation can be selected. We will input to the amount of water vapor on the ground, etc. at the selected location to confirm how the meteorological field can be changed, and whether the field can be changed significantly with a smaller input compared to the case with randomly selected locations or locations determined based on physical insight. In addition, by investigating the mechanism by which the meteorological field changes due to input, it is expected that more effective input methods will be considered.

