R&D Theme

Mathematical Problem Formulation for Actuator Location Optimization

Progress until FY2022

1. Outline of the project

Actuator-position optimization problems are formulated to find actuator positions that efficiently change states such as the temperature and the velocity of the atmosphere. By clarifying the evaluation metrics, it becomes possible to quantitatively evaluate the effect of the actuator placement and to develop an algorithm to determine the actuator placement based on this evaluation index. If the formulated mathematical problem is too complex, it will be difficult to solve this problem for actuator placement in such a large degree-of-freedom field.

To formulate the actuator-placement problems, we first focus on sensor-placement problems for linear models, which have been studied in many previous works. We formulate the actuator-placement problem by deriving a duality of the existing sensor-placement problem for linear models (Figure 1). By using duality, the sensor placement algorithm can be converted to the actuator placement algorithm with only minor modifications, and thus, the development cost for it can be reduced. However, the actuator placement problem obtained by using duality must be interpreted, since the physical and engineering meanings of such problem is not clear. The knowledge gained from the formulation of the actuator-placement problems will be used to formulate the actuator-placement problems for nonlinear models, which will be important for application to weather fields.

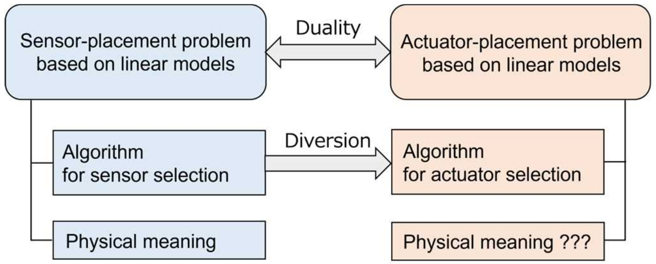

Figure 1: Formulation of actuator-placement problem using the duality

2. Outcome so far

Actuator-placement problems for linear models were formulated using the duality. By considering the duality of the sensor-placement problem for optimal state estimation for linear dynamic models we derived the actuator-placement problem for optimal control for linear dynamic models (Figure 2). The matrices Q and R that appear in both placement problems refer to the covariance matrices of the noises in the sensor-placement problem, whereas, in the dual actuator-placement problem, they refer to the weight matrices in a control evaluation metric, in other words the matrices associated with the states and inputs that we regard as important. Since actuators with the same properties will be assumed to be used in the control of the weather, it is natural to set the input weight matrix R to be a unitary matrix. The setting of the unitary matrix reduces the computation time required to normalize the matrix when actuator placement is implemented. This actuator-placement problem can be solved approximately by modifying the sensor-placement algorithm that has been developed in another theme.

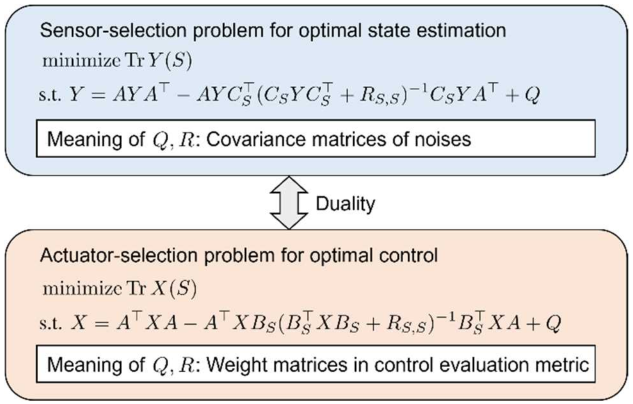

Figure 2: Dual placement problems for linear systems

3. Future plans

We begin to formulate actuator-placement problems for nonlinear systems. For this purpose, we focus on the singular vector method, which is often used in the field of meteorology. The singular vector methods deal with problems that aim to maximize the variation of the terminal state with respect to the variation of the initial state. By extending this method, we formulate a problem that aims to maximize the variation of the state trajectory with respect to the input to actuators (Figure 3). This allows more appropriate evaluation of actuator placement in the weather field.

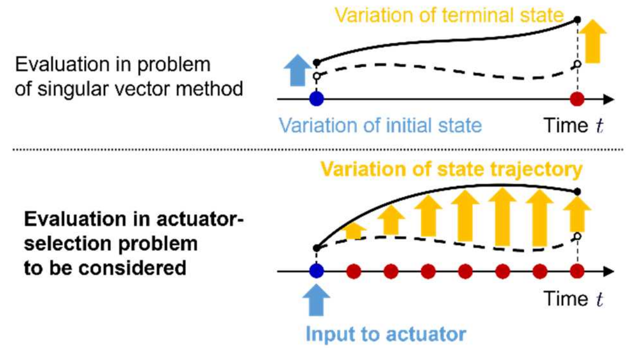

Figure 3: Actuator-placement problem to be considered