

5. Mathematical Approach

Progress until FY2024

1. Outline of the project

We primarily aim to identify the optimal timing and location for interventions in tropical cyclone (TC) control, based on theory and numerical models.

To achieve this, we are pursuing two parallel approaches: one involves introducing the cutting-edge findings from the mathematical sciences to elucidate mathematical structures that allow small external forces to induce significant changes; the other applies adjoint-based operators corresponding to realistic physical models to trace back and identify the physical quantities that have a major impact on wind speed and precipitation of a TC.

In addition, by collaborating with other research and development projects, we also contribute to reducing uncertainties through data assimilation, a technique that integrates observational data with numerical models.

2. Outcome so far

Our project was newly launched in January 2025 under the theme (1) to clarify mathematical structures that enable significant controlling effects by a small input. In April 2025, another PI in meteorological approaches transferred to this project for the theme (2) to derive optimal perturbations for control based on advanced applications of adjoint operators.'

Theme (1) aims to apply cutting-edge knowledge from mathematical sciences to simple systems in order to understand mechanisms by which small external forces (perturbations) can induce significant changes, and to ultimately apply these findings to the large-scale and complex TC system.

Until the 20th century, research in the dynamical theory known as uniformly hyperbolic dynamical systems was actively investigated and led to substantial advances in understanding. However, such systems cannot adequately represent mathematical structures involved in TC, such as intermittency or multi-scale interactions. Therefore, incorporating the latest

theoretical advances is essential.

In the fiscal year 2024, we conducted research using a mathematical model known as the Hénon map which exhibits chaotic behavior. The results revealed that introducing perturbations perpendicular to the local stable manifold—a set of points that converge to periodic points as time goes to infinity—can lead to significant changes in the future (Figure 1).

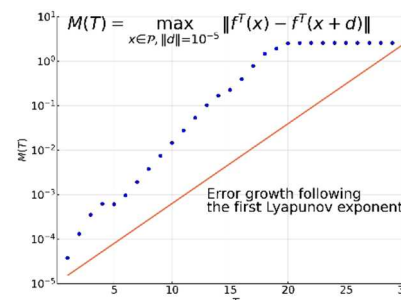


Figure 1. Magnitude of perturbations at each time step. The orange line represents growth based on the Lyapunov exponent of the chaotic dynamics, while the blue dots represent the most efficiently growing perturbations.

Theme (2) aims to identify robust and rapidly-evolving perturbations through the advanced use of adjoint method. We will develop the methodologies toward the application to the realistic TCs.

The adjoint method involves selecting a specific target—such as TC intensity or rainband precipitation—and tracing back in time, under a linearity assumption whose important characteristics is the output changes proportional to input changes, the physical variables and locations that have a significant influence on the target. For example, it allows us to investigate which variables in which region should be perturbed to optimally alter the kinetic energy near the TC center two days later.

By the end of fiscal year 2023, the physical model, known as WRF (Weather Research and Forecasting model), as well as the

adjoint operator, had been ready for use. However, for forecasting TCs more than several hours ahead, the linearity assumption does not hold true, so special treatments were required

In fiscal year 2024, we targeted the kinetic energy near the center of TC Nanmadol (2022), simulated using a 5-km mesh model. By repeatedly performing one-hour optimizations—where linear behavior is expected—and altering water vapor hourly, the intensity of the cyclone could be successfully reduced (Figure 2). However, since larger impacts may be achievable within a nonlinear framework, we have also initiated work on a method known as nonlinear optimal perturbation.

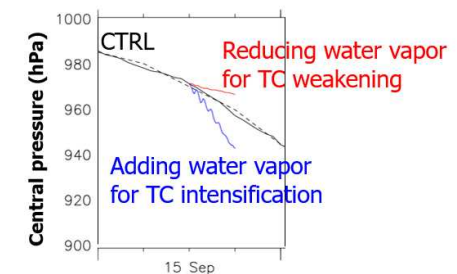


Figure 2. Simulated central pressure of TC Nanmadol by adding and reducing water vapor where high-impact for TC intensity change was expected by an adjoint operator.

3. Future plans

The current results imply that avoiding perturbations in stable directions can lead to greater changes than conventional methods. On the other hand, in order to apply this to complex problems such as TC control, it will be necessary to consider appropriate dimensional reduction techniques and the introduction of new computational methods.

Regarding the application of adjoint equations to realistic tropical cyclones, high-resolution experiments using a 1-km mesh can more effectively alter cyclone-associated energy and precipitation. We are also actively working on deriving nonlinear optimal perturbations to identify physical quantities and regions that could induce even greater changes than the current ones.