FY2022 JST-RIKEN Joint AIP Open Symposium

Efficient Machine Learning with Tensor Networks

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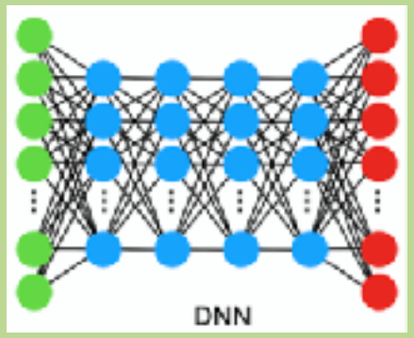
Trends of Machine Learning

Big Data

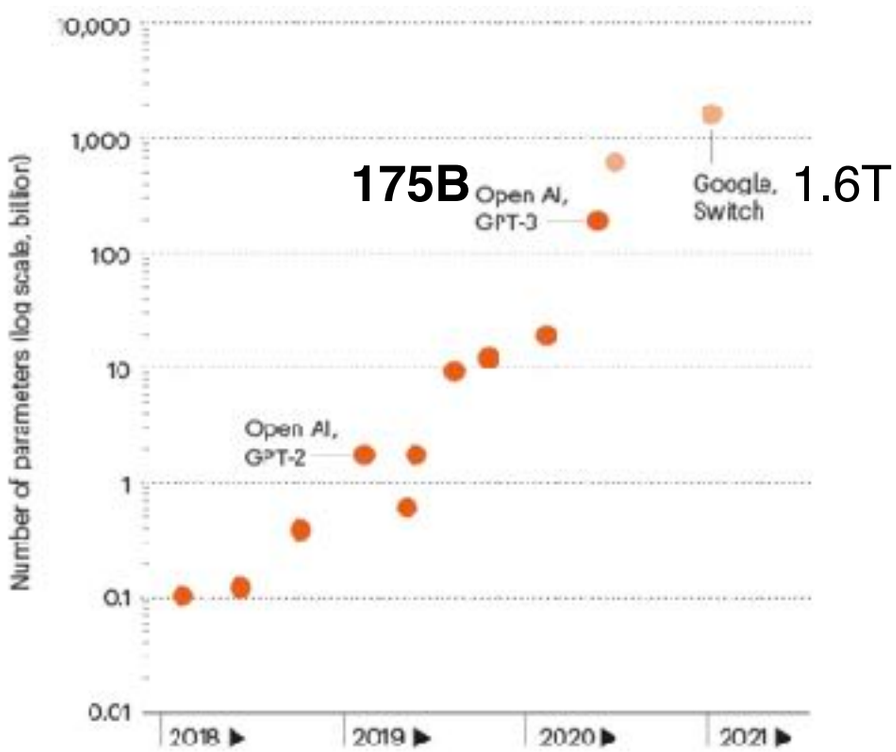
Large Model



Computation

OpenAI's GPT-3

Dataset: 45 TB text data



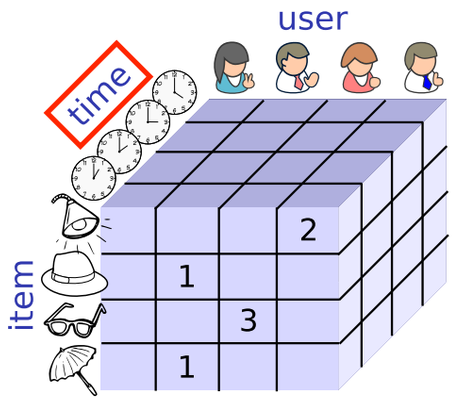
https://www.nature.com/articles/d41586-021-00530-0

OpenAI's GPT-3

- 28 TFLOPS V100
- 355 GPU years
- $4.6 M

Challenges from data perspective

- Learning knowledge from incomplete & limited data, noisy data, or adversarial corrupted data



3 order tensor

Recommender system

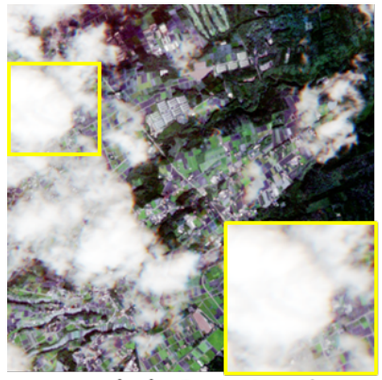
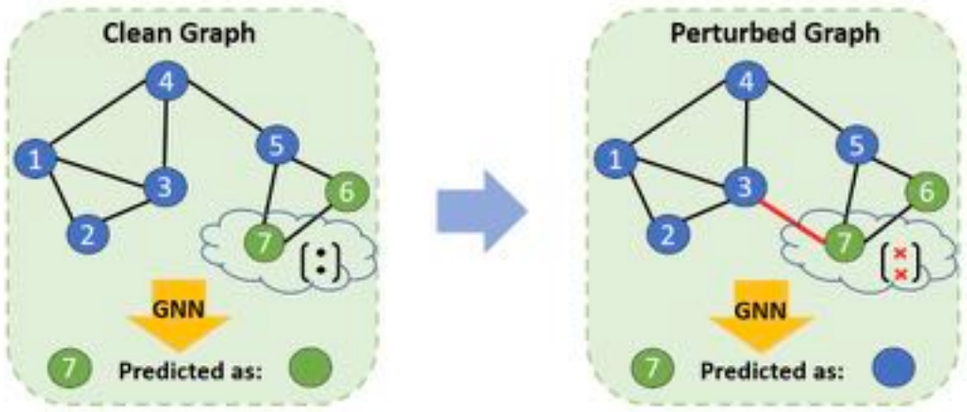


Image inpainting/denoising

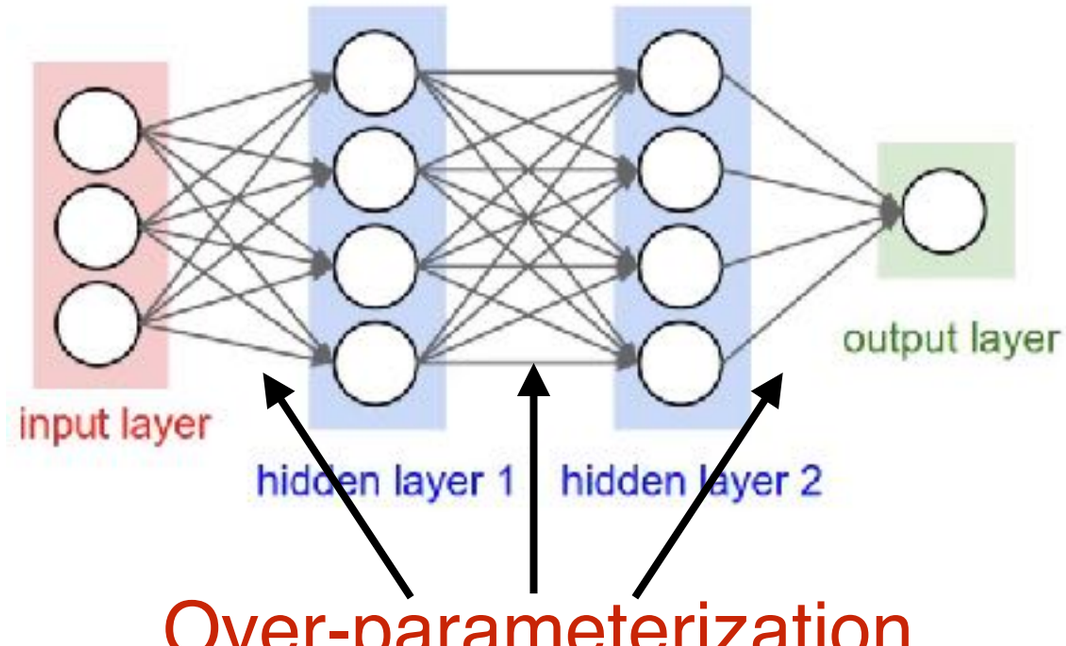
graph prediction



[Jin et al. SIGKDD 2021]

Poisoning or adversarial attack

Challenges from model perspective

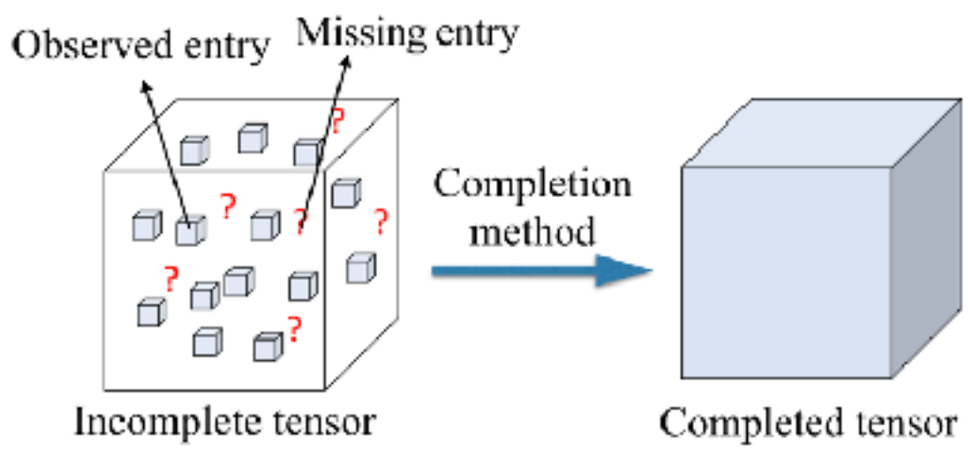


Over-parameterization

- Complex architecture, large number of parameters, heavy computation for training and inference.
- Lack of interpretability and lack of robustness to adversarial attacks.
- How to dramatically increase model capacity without significant increasing of model size?

Multi-dimensional, Incomplete and Noisy Data

- Task: learning from limited tensor entries to predict unobserved entries



$$\mathcal{Y}_\Omega \to \mathcal{Y}_{\bar{\Omega}}$$

- Challenges:
  - Data efficiency
  - Scalability and efficient optimization algorithms
  - Exact recovery guarantee

Tensor Completion

Objective:

$$\min_{\mathcal{X}} \|\Omega * (\mathcal{Y} - \mathcal{X})\| + R(\mathcal{X})$$

Fitting error Structure Regularizer

Popular approaches:

- Low-rankness assumption (convex, not scalable)

$$R(\mathcal{X}) = \|\mathcal{X}\|_*$$

- Decomposition based approach (optimal rank selection)

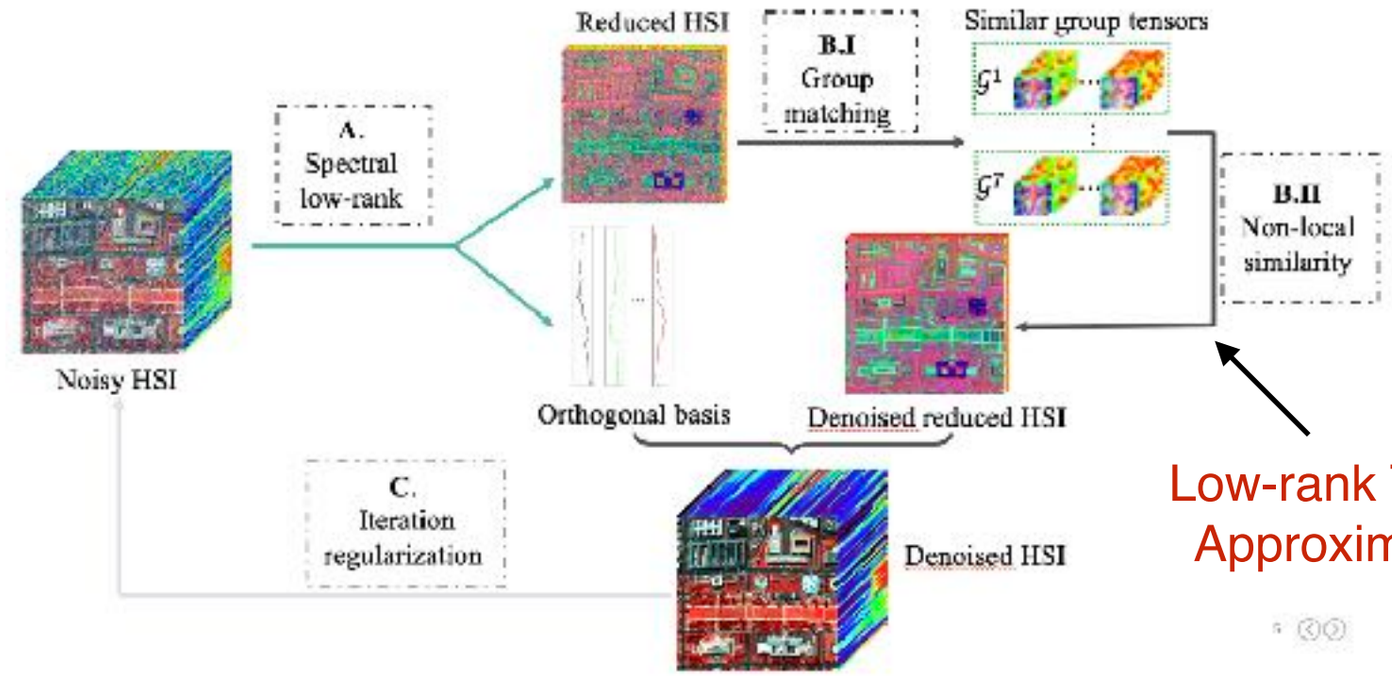
$$R(\mathcal{X}) = \|\mathcal{X} - \mathrm{TN}(\mathcal{G}_1, \ldots, \mathcal{G}_N)\|$$

- Prior knowledge (smoothness, non-negative), side information

Low-rankness under Linear Transformation

- **Image Denoising**: large scale image is not globally low-rank

(He et al., CVPR 2019)



(Li et al, CVPR 2019)

- **Non-uniform missing patterns** (slice, fiber missing)

$$\min_{\mathbf{X}\in\mathbb{R}^{m_1\times m_2}} \|\mathcal{Q}(\mathbf{X})\|_* \quad s.t. \|\mathcal{P}_\Omega(\mathbf{X}) - \mathcal{P}_\Omega(\mathbf{Y})\|_F \leq \delta,$$

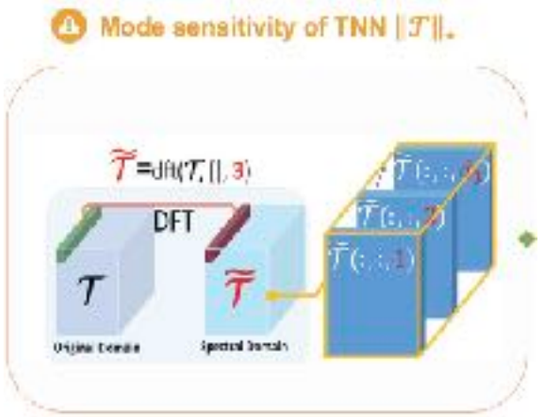
Linear transformation

Error bound is theoretically guaranteed

Enhanced low-rank modeling for tensor SVD

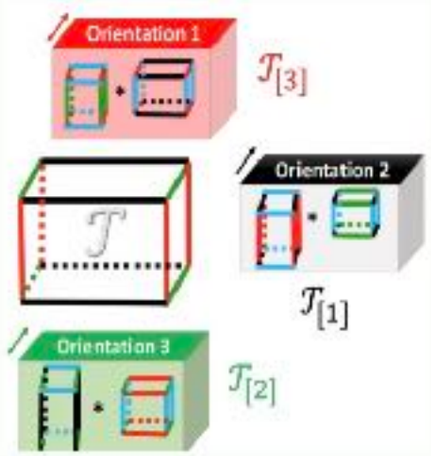
(A. Wang et al., AAAI 2020)

- **Problem:** t-SVD has mode sensitivity.
- Two **mode invariant tubal nuclear norms** with error bound



Two mode invariant TNNs

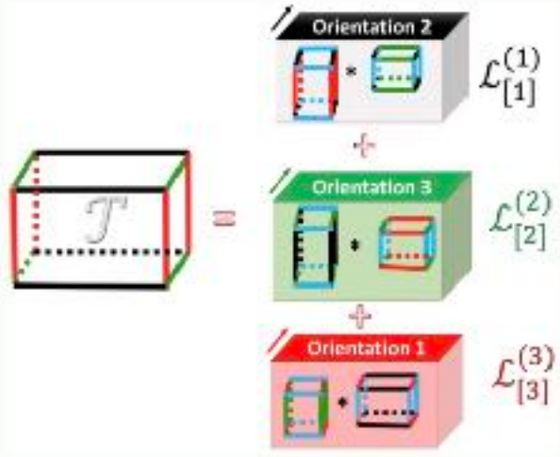
$$\|\mathcal{T}\|_{\text{overlap}} = \sum_{k=1}^{K} \|\mathcal{T}_{[K]}\|_*$$



simultaneously low-tubal-rank in all modes

$$\|\mathcal{T}\|_{\text{latent}} = \min_{\mathcal{T}=\sum_{k=1}^{K}\mathcal{L}^{(k)}} \sum_{k=1}^{K} \left\|\mathcal{L}^{(k)}_{[k]}\right\|_*$$

sum of latent low-tubal-rank tensors



$$\frac{\|\mathcal{L}^* - \hat{\mathcal{L}}_{\text{overlap}}\|_F^2}{d^K} \le C_1\sigma^2\left(\|\mathcal{S}^*\|_0 K\log d + d^{-1}K^{-2}\sum_k r_t(\mathcal{L}^*_{[k]})\right)$$

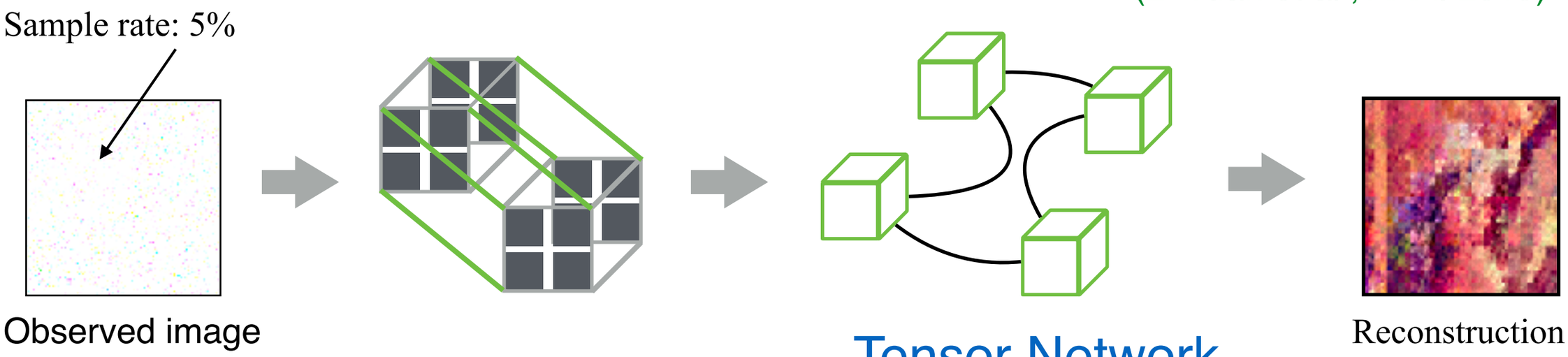
error bounded in sum of tubal ranks in all modes

$$\frac{\|\mathcal{L}^* - \hat{\mathcal{L}}_{\text{latent}}\|_F^2}{d^K} \le C_2\sigma^2\left(\|\mathcal{S}^*\|_0 K\log d + d^{-1}\min_k r_t(\mathcal{L}^*_{[k]})\right)$$

error bounded by mode of minimal tubal rank

Tensor Networks with Low-rank Cores

(L. Yuan et al., AAAI 2019)



Fitting error — Nuclear norm on core tensor — TT/TR decomposition

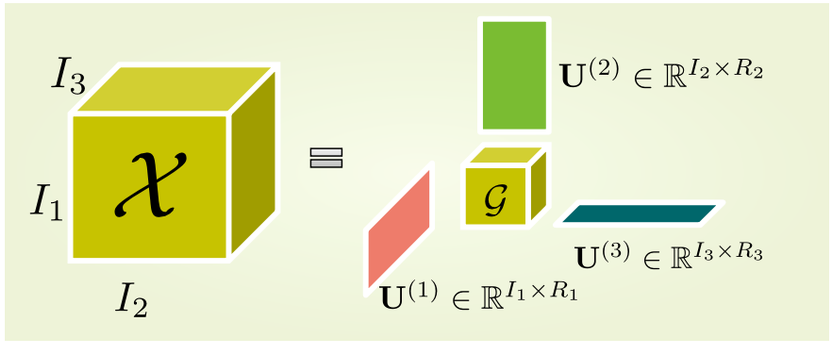
$$\min_{\mathcal{G}} \quad \left\| \Omega * (\mathcal{Y} - \hat{\mathcal{Y}}) \right\|_F^2 + \lambda \sum_{n=1}^{d} \sum_{i=1}^{3} \left\| \boldsymbol{G}_{(i)}^{(n)} \right\|_*, \quad s.t. \quad \hat{\mathcal{Y}} = \mathrm{TR}(\mathcal{G}^{(1)}, \cdots, \mathcal{G}^{(d)}).$$

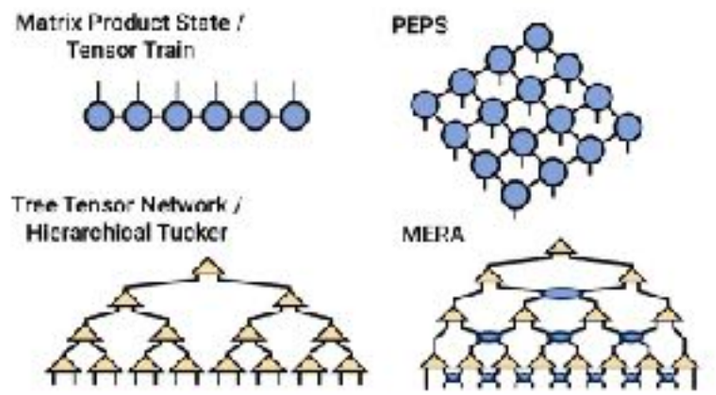
- Tensorization allows for capturing complex structural dependency
- Efficient optimization by combining decomposition and nuclear norm minimization

What is Tensor Network?

2-order → 3-order → *N*-order

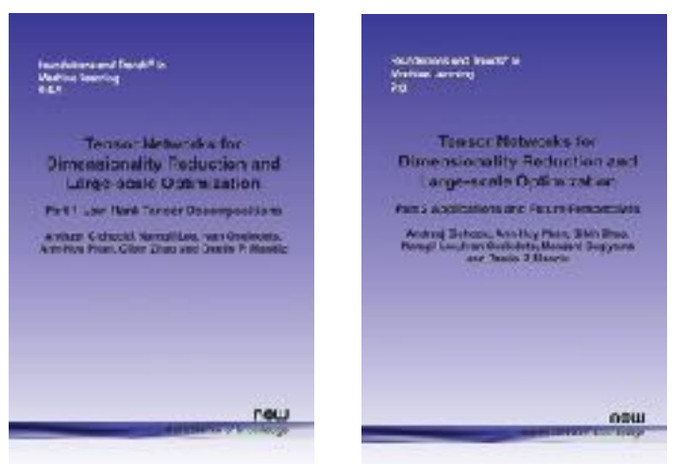
Matrix Factorization → Tensor Factorization → Tensor Networks





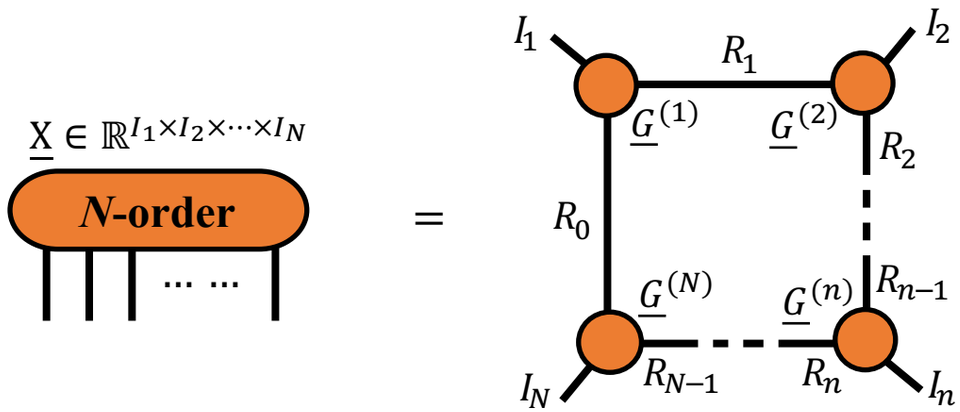
https://tensornetwork.org

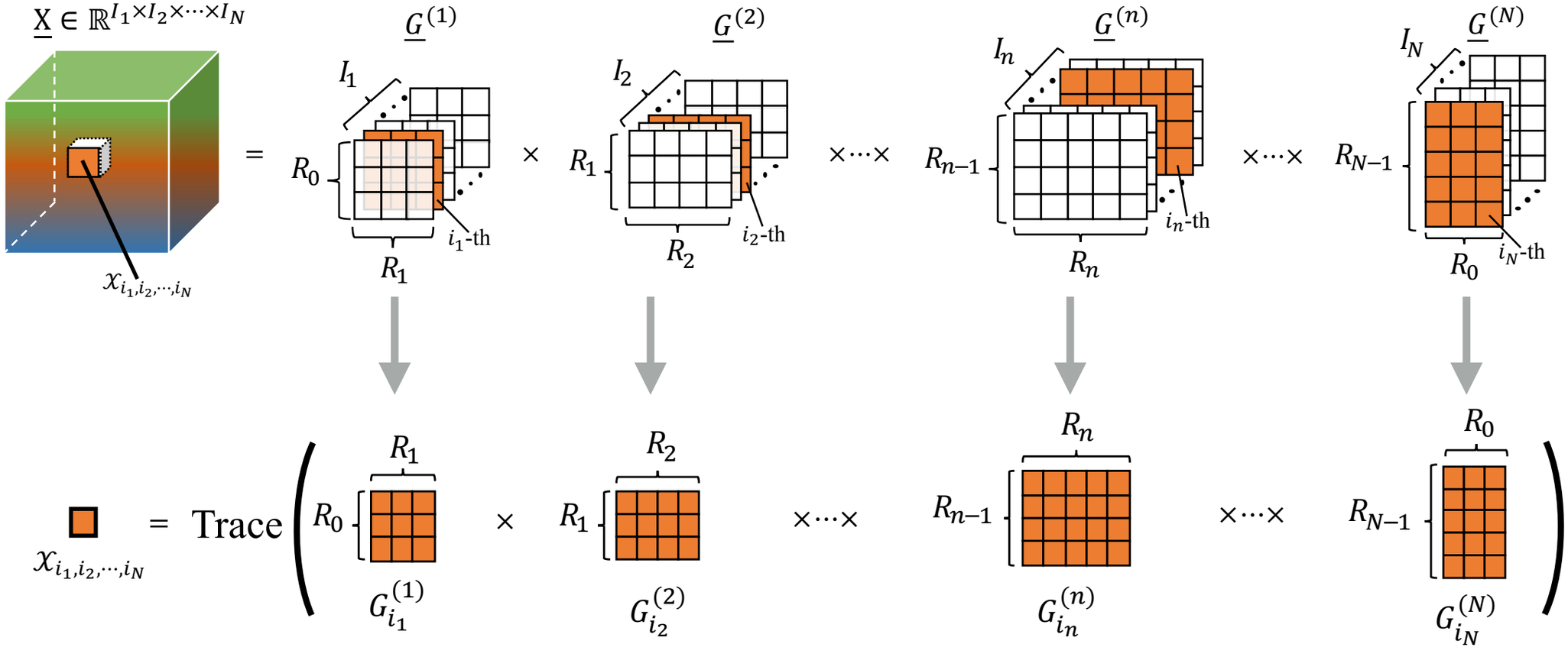
- Representation of *N*-order tensor as contractions of O(*N*) smaller tensors
- Physics: to describe entangled quantum many-body systems



Tensor Ring Decomposition

(Zhao et al., arXiv 2016, ICASSP 2019)



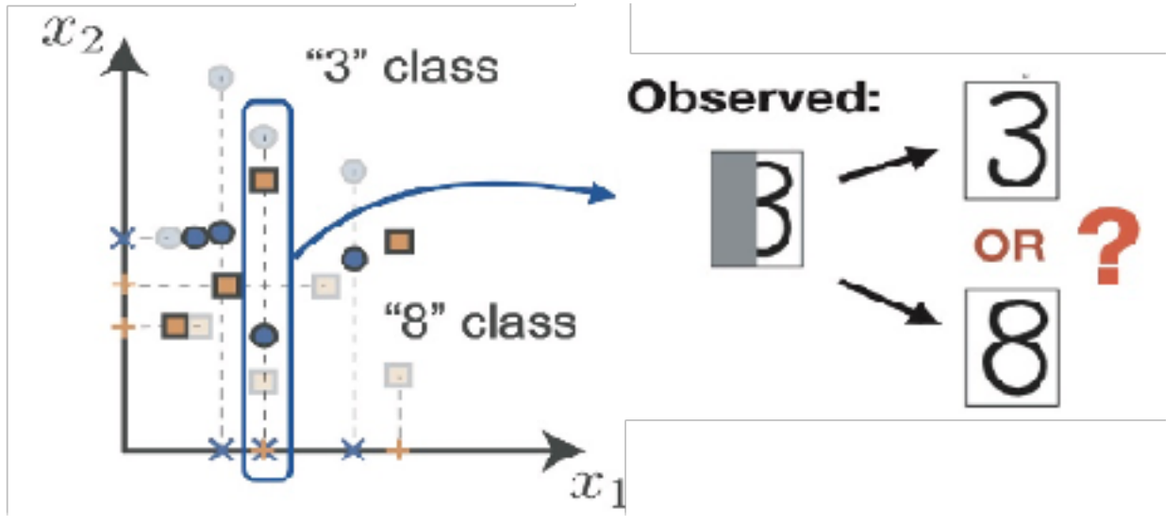


Classification of incomplete data

Problem: learning classification model from incomplete data $(x_n^{miss}, y_n), n = 1,\ldots,N$

Objective: $\hat{f}(g(x^{miss}), \hat{\theta}) \approx f(x, \theta)$

Reconstruction of incomplete data



Sequential approach (completion + classification)

- Cannot ensure statistical consistence of classifier
- Exact recovery is not guaranteed because label information is ignored

Simultaneous reconstruction and classification

(Caiafa et al., CVPR workshop 2021)

- Learning sparse representation and classifier collaboratively (NNs + sparse coding)

$$J(\Theta, \mathbf{D}, \mathbf{s}_i) = \frac{1}{I}\sum_{i=1}^{I}\left\{J_0(\Theta, \hat{\mathbf{x}}_i, y_i) + \lambda_1 J_1(\mathbf{D}, \mathbf{s}_i) + \lambda_2 J_2(\mathbf{s}_i)\right\}$$

Classification loss (e.g. crossentropy) for any classifier (deep network)

Representation error

$$J_1(\mathbf{D}, \mathbf{s}_i) = \frac{M}{N}\|\mathrm{m}_i * (\mathbf{x}_i - \mathbf{D}\mathbf{s}_i)\|^2$$

Promotes sparsity

$$J_2(\mathbf{s}_i) = \frac{1}{N}\|\mathbf{s}_i\|_1$$

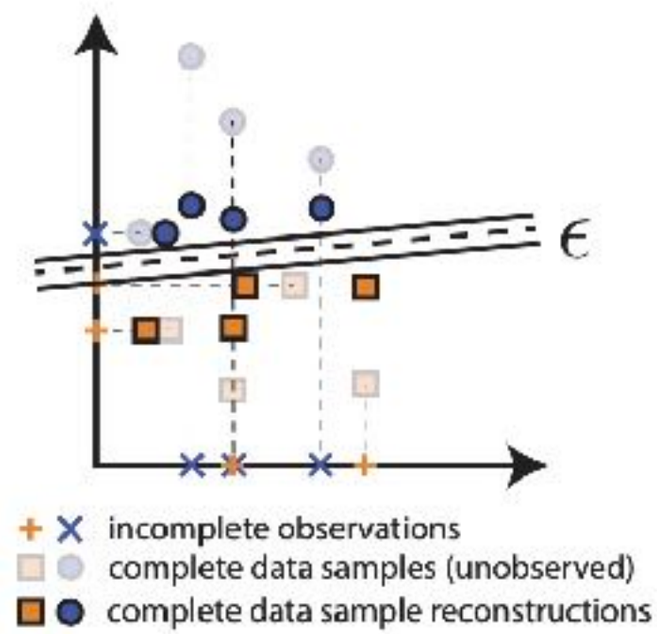
- Sufficient condition

Weights of classifier

$$\epsilon > |\langle \mathbf{w}^m, \mathbf{x}^m \rangle| + |\langle \mathbf{w}^m, \hat{\mathbf{x}}^m \rangle|$$

Original data (missing part)

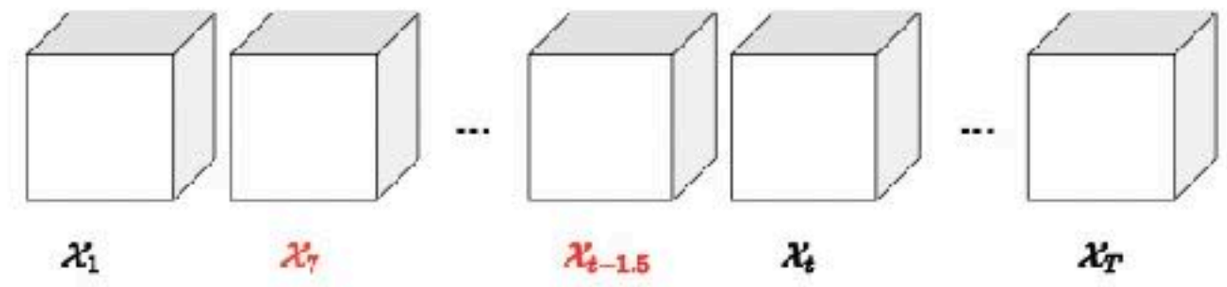
Reconstructed data (Missing part)



Time series data with missing time points

Task: Given tensorial time series with **irregular/missing time steps**, to train a model for prediction on **continuous time points**.

Examples: videos with missing frames, relations between stock market prices of many companies, etc



Challenges:

- **Tensorial NN/RNN** (Bai et al. 2017): Incapable of handling irregular time steps, and prediction on decimal time points
- **Neural ODE** (Chen et al. NeurIPS 2018): Ignoring spatial structure information, large number of parameters

Tensor Neural ODE

(Bai et al., IJCNN 2021)

We directly process the tensorial time series $\{\mathcal{Y}_t\}_{t\in[0,T]}, \mathcal{Y}_t \in \mathbb{R}^{I_1\times\cdots\times I_N}$, proposing tensor neural ODE (TENODE)

$$\frac{\mathrm{d}\mathcal{Y}(t)}{\mathrm{d}t} = f_{\Theta}(\mathcal{Y}(t), \mathcal{X}(t), t)$$

with the control input $\mathcal{X}(t)$ and the initial condition $\mathcal{Y}(0) = \mathcal{Y}_0$. Parameter size: from $O(I^{2N})$ of neural ODE to $O(NI^2)$

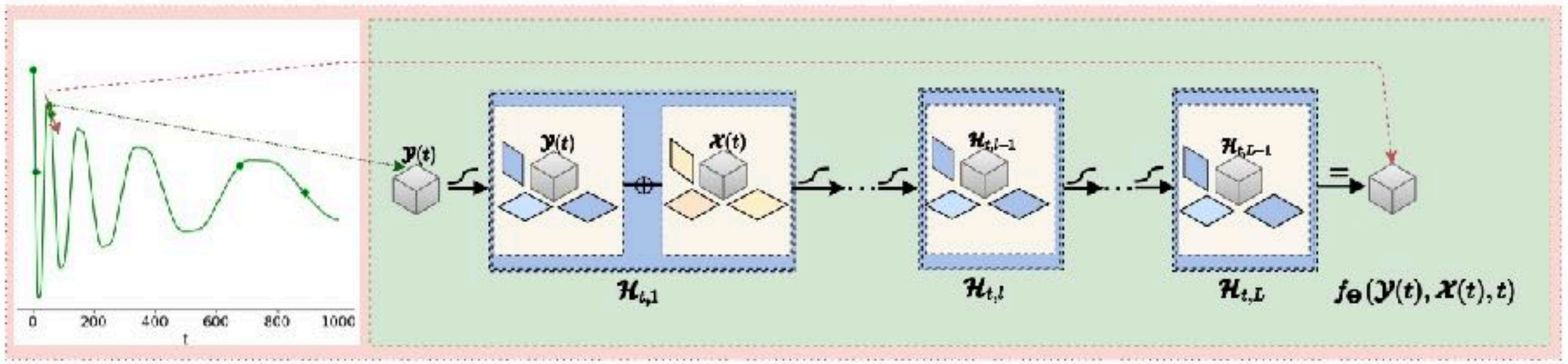
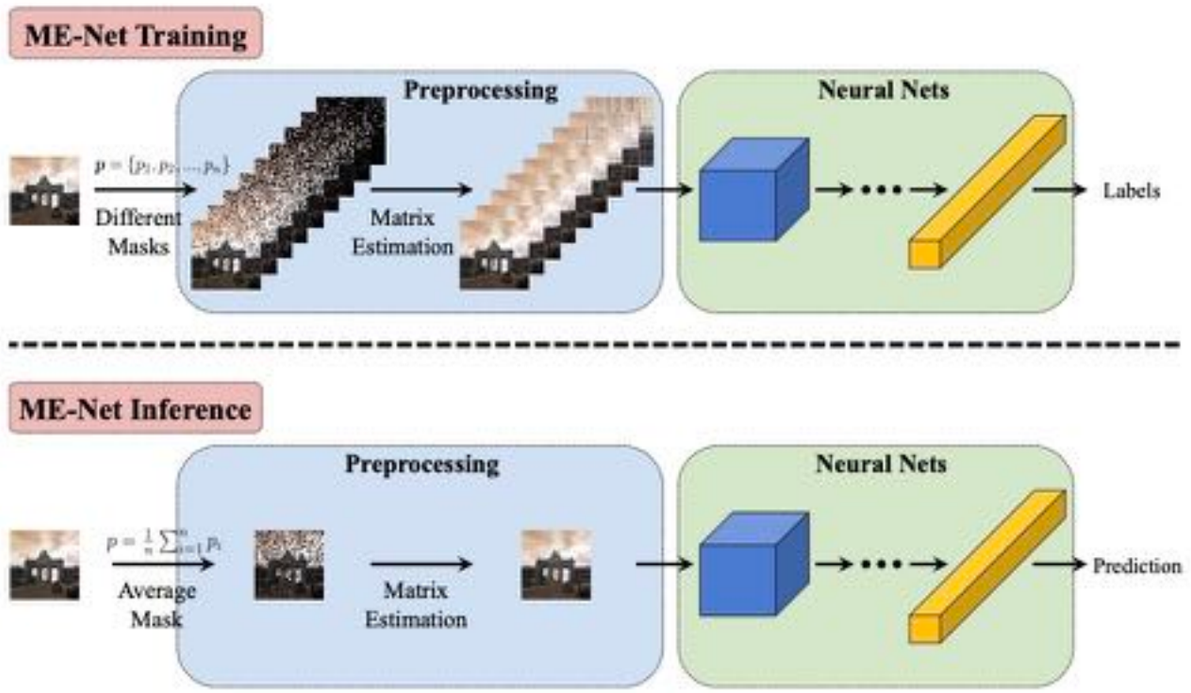


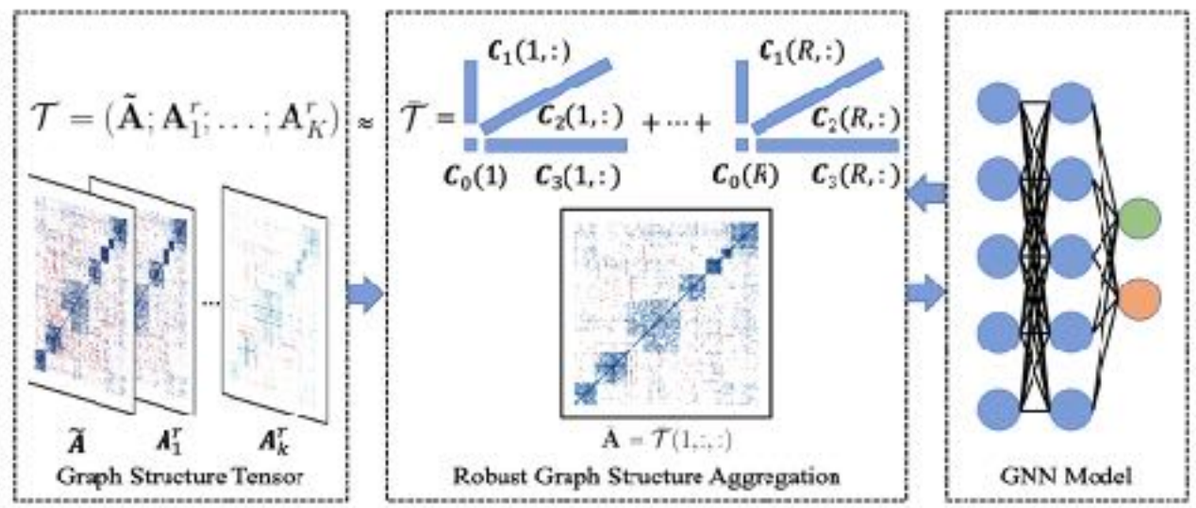
Figure 5: Architecture Overview: Tensor neural ODE (TENODE)

Removing adversarial perturbations from data

- Tensor completion can destroy adversarial perturbations [Yang et al. ICML 2019]

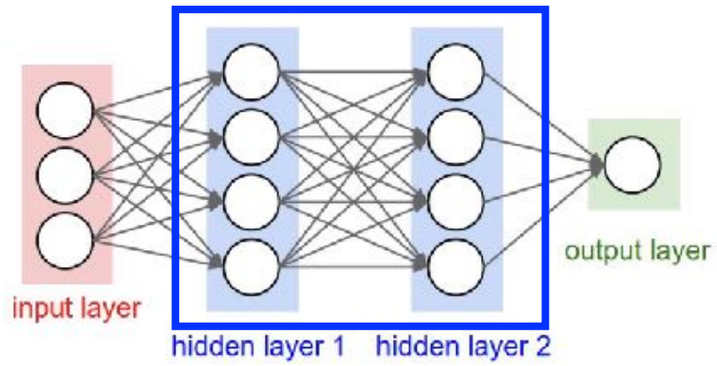


- Defending GNNs via tensor-based robust graph aggregation



Parameter efficient modeling via Tensor Networks

Model Compression



$$\boldsymbol{h}_2 = \sigma(\underset{(N \times M)}{\boldsymbol{W}^T} \boldsymbol{h}_1 + \boldsymbol{b})$$

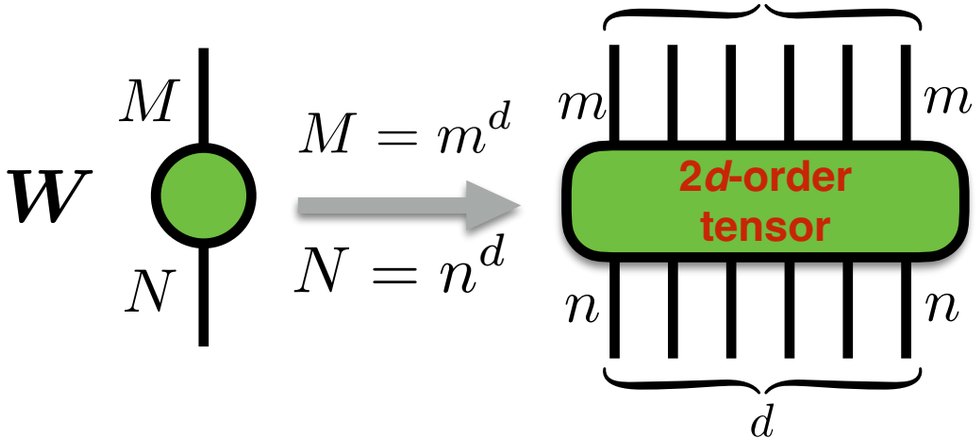
Hidden layer (output)

Hidden layer (input)

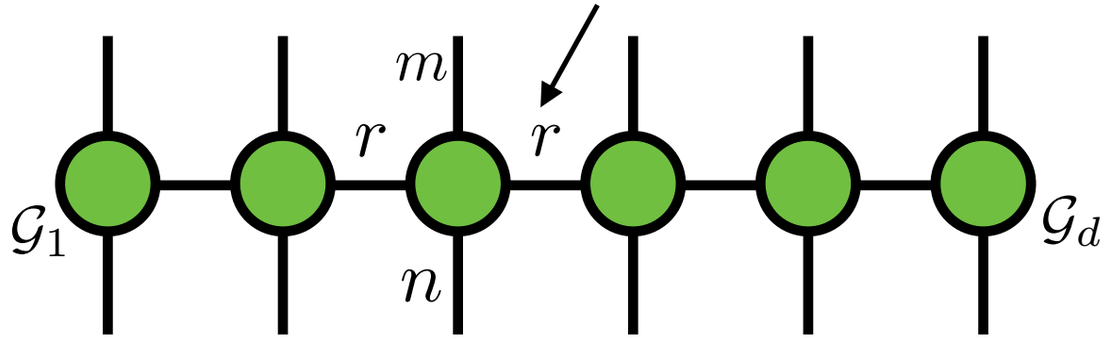
Weights

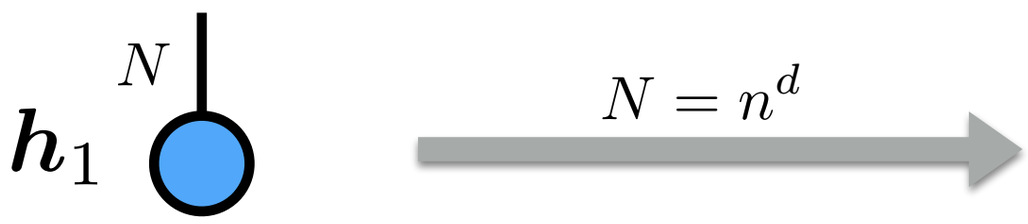
Number of parameters $d\sqrt[d]{MNr^2}$

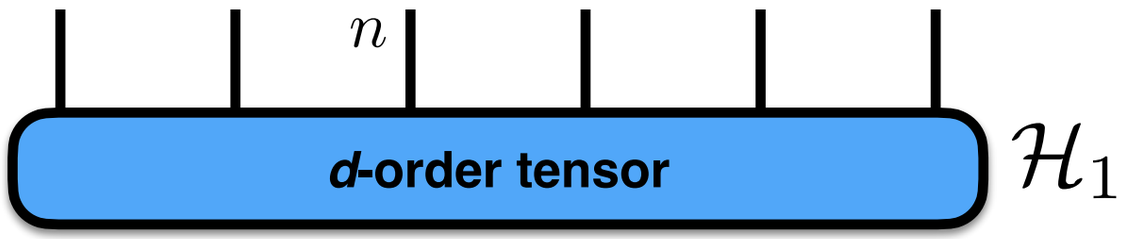
Tensorization



TT Matrix







$$\boldsymbol{W}^T\boldsymbol{h}_1 = \mathrm{TT}(\mathcal{G}_1, \cdots, \mathcal{G}_d) \times_{1,\ldots,d} \mathcal{H}_1$$

[Novikov et al., NeurIPS 2015]

Higher-order latent factor analysis

(Tao et al., ACML 2021)

$$\boldsymbol{y} = \boldsymbol{W}\boldsymbol{\eta} + \boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}),$$

- Given higher-order data $\mathcal{Y} \in \mathbb{R}^{P_1 \times \cdots \times P_D}$, marginalize η gives $\mathcal{Y} \sim \mathcal{N}(0, \mathcal{V})$

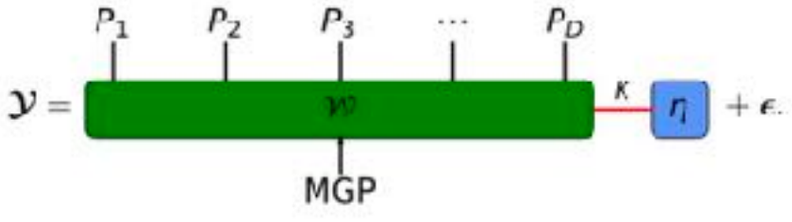
Covariance of vectors: $\boldsymbol{V}_{ij} = \mathrm{cov}(\boldsymbol{y}_i, \boldsymbol{y}_j)$.

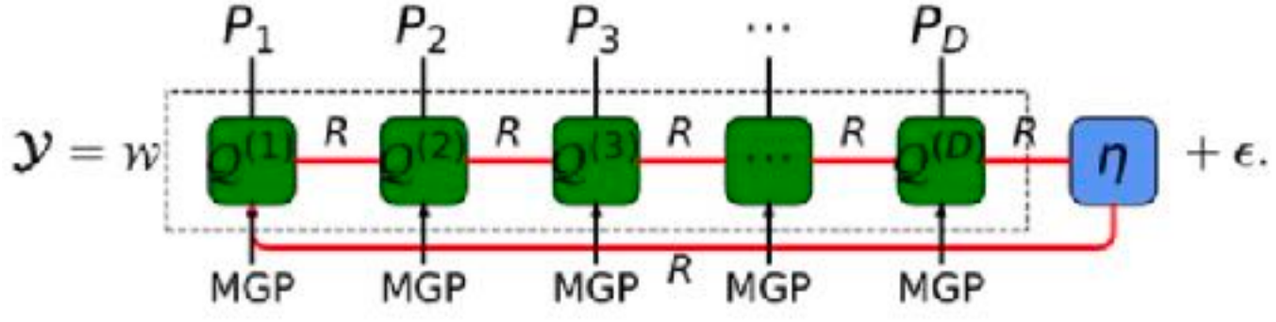
Covariance of tensors: $\mathcal{V}_{i_1 i_2 i_3 j_1 j_2 j_3} = \mathrm{cov}(\mathcal{Y}_{i_1 i_2 i_3}, \mathcal{Y}_{j_1 j_2 j_3})$.

Core tensors

$$\mathcal{V}_{p_1 \cdots p_D p'_1 \cdots p'_D} = \underbrace{\mathrm{tr}(\boldsymbol{Q}^{(1)}[p_1] \cdots \boldsymbol{Q}^{(D)}[p_D](\boldsymbol{Q}^{(D)}[p'_D])^\intercal \cdots (\boldsymbol{Q}^{(1)}[p'_1])^\intercal)}_{\text{low-rank TR}} + \underbrace{\tau^{-1}}_{\text{noise}},$$

- TN representation of parameter W





$$\mathcal{Y} = \ll \mathcal{Q}^{(1)}, \ldots, \mathcal{Q}^{(D)}, \eta \gg + \mathcal{E},$$

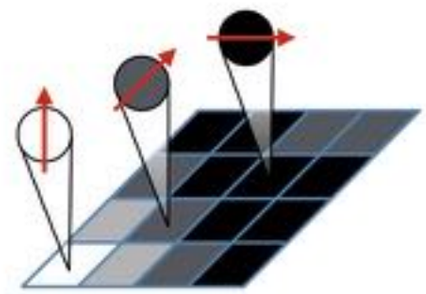
TN representation of inputs

- Mapping input data into TN representation

$$\mathbf{x} = [x_1, x_2, \ldots, x_d]^T$$

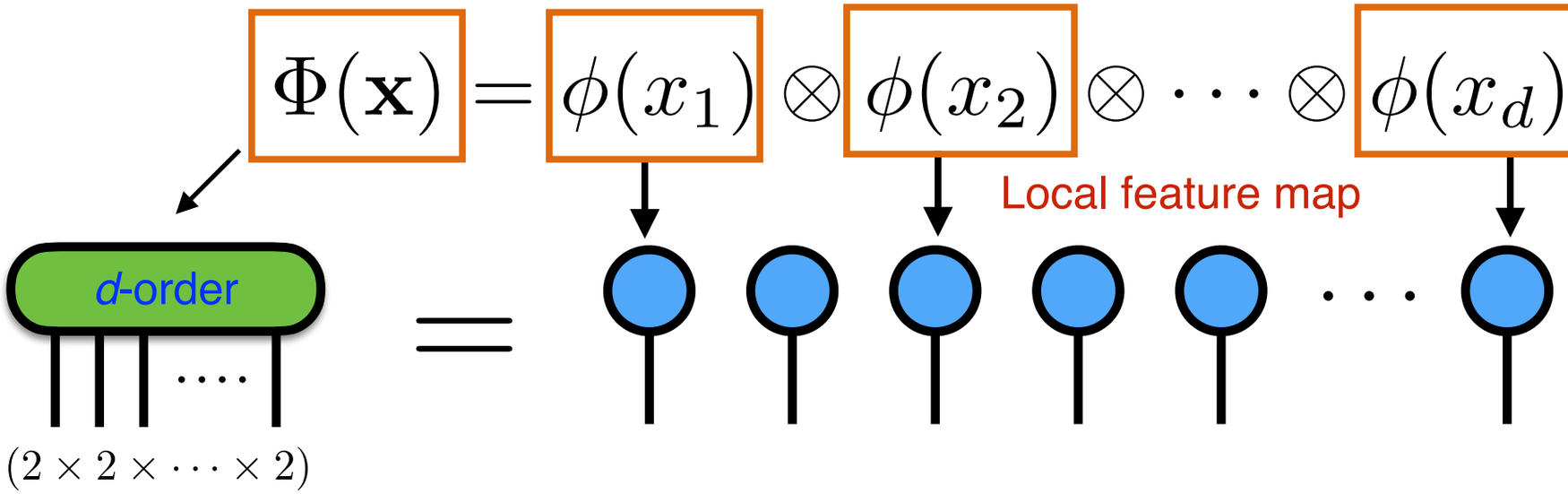
$$\phi(x_i) = \left[\cos\left(\frac{\pi}{2}x_i\right), \sin\left(\frac{\pi}{2}x_i\right)\right]^T$$

Inspired by “spin” vectors in quantum system



$$\Phi(\mathbf{x}) = \phi(x_1) \otimes \phi(x_2) \otimes \cdots \otimes \phi(x_d)$$

Local feature map



Rank-1 Tensor

$$d \mapsto 2^d$$

- Accuracy of 99.03% on MNIST by one layer

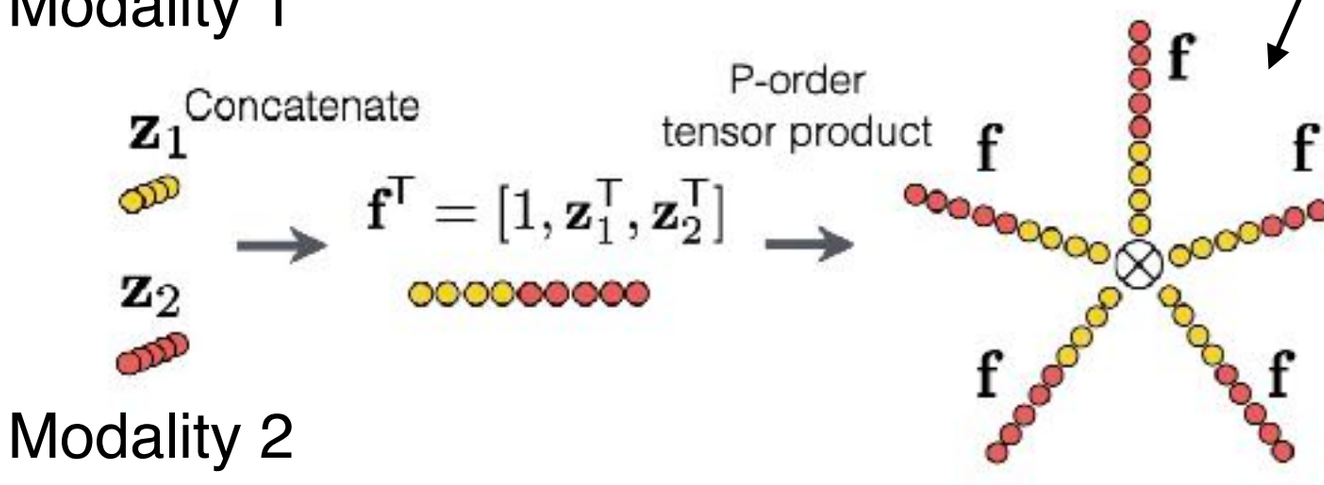
Supervised Learning with Quantum-Inspired Tensor Networks [Stoudenmire et al., NIPS 2016]

High-order Tensor Fusion

(Hou et al., NeurIPS 2019)

- Tensor Polynomial Pooling (PTP)

$$\mathcal{F} = \underbrace{\mathbf{f} \otimes \mathbf{f} \otimes \cdots \otimes \mathbf{f}}_{\text{P-order}}$$



Feature Interactions

- Linear
- Bilinear
- Trilinear
- Intra-modal
- High-order

Dimensionality increases exponentially with *P*

$$(md)^P$$

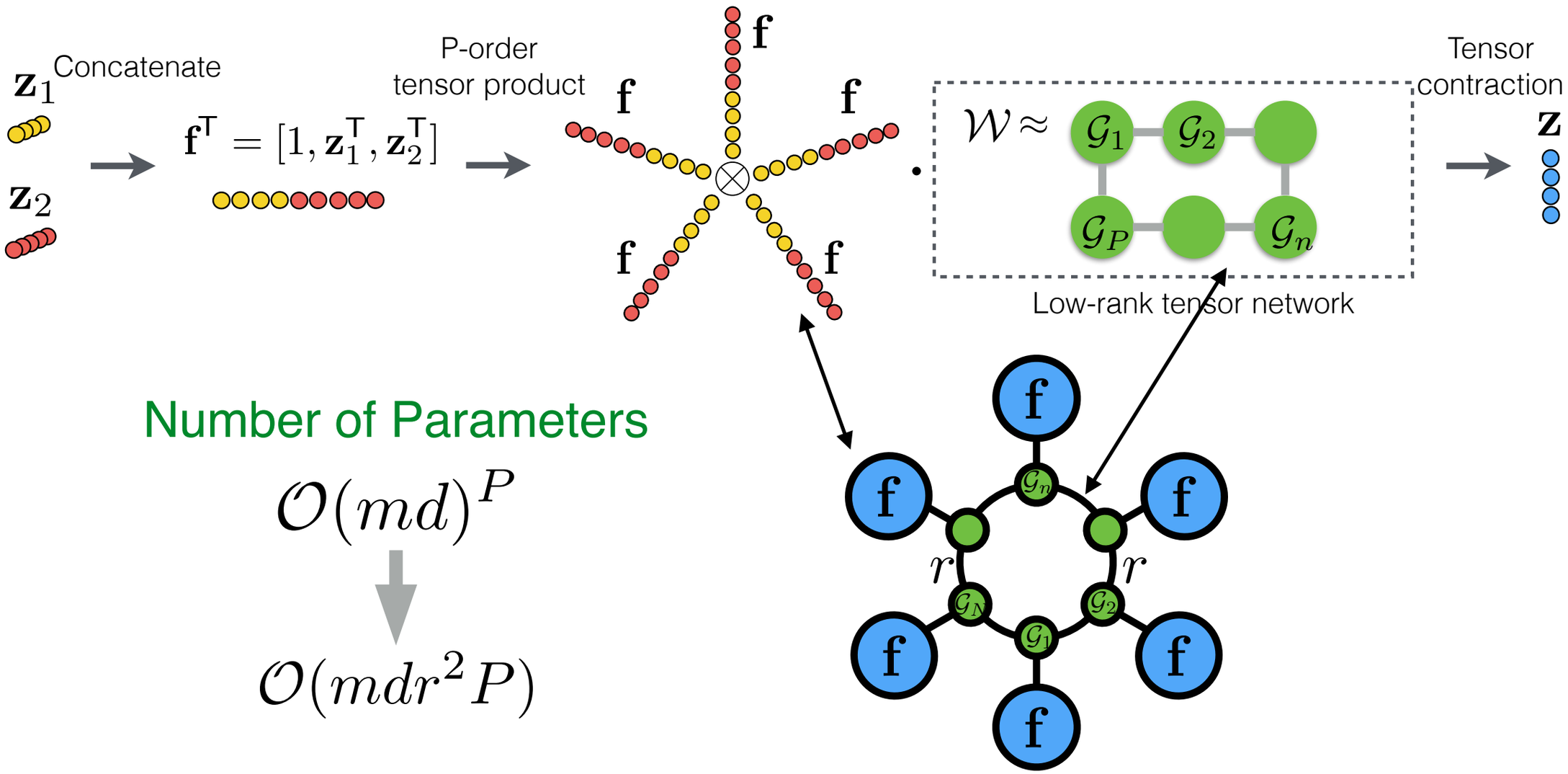
Order

Number of modality

Dimension of feature

Tensor Polynomial Pooling (PTP)

(Hou et al., NeurIPS 2019)



Number of Parameters

$$\mathcal{O}(md)^P$$

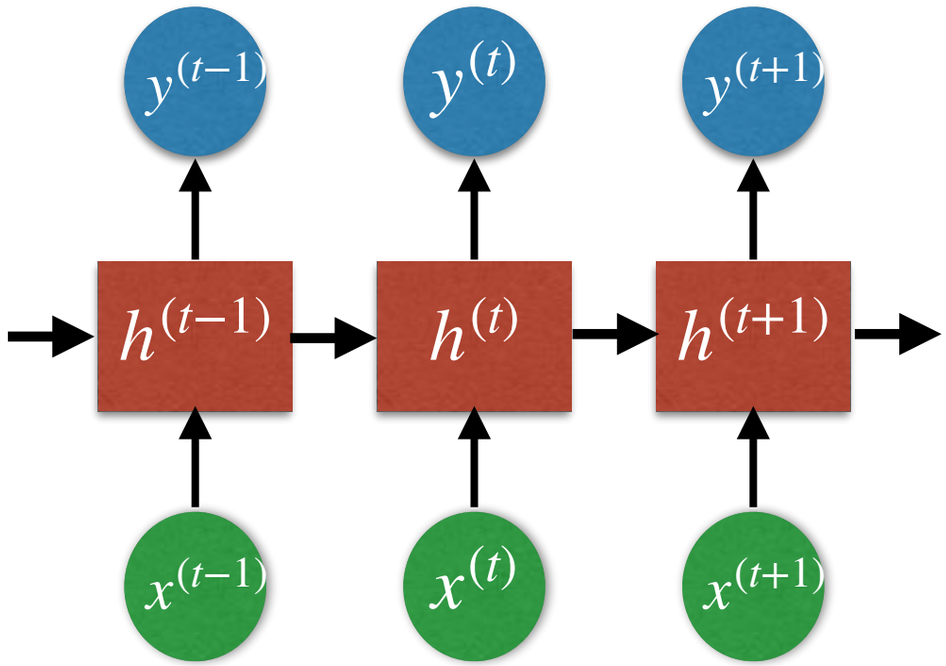
$$\mathcal{O}(mdr^2P)$$

Polynomially enhanced capacity with linearly increasing number of parameters

Tensor-Power Recurrent Models

(Li et al., AISTATS 2021)

- RNN and LSTM do not have long memory from a statistical perspective [Zhao et al., ICML 2020]



RNN

$$h^{(t)} = \sigma(Wh^{(t-1)} + Ux^{(t)} + b)$$

Transition function

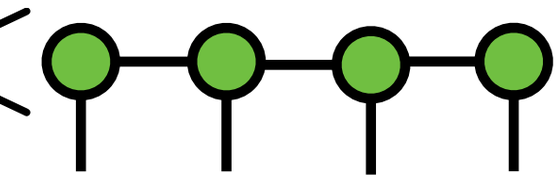
$(p+1)$-order weight tensor

$$\mathbf{h}^{(t)} = \mathcal{G} \underbrace{\times_1 \begin{pmatrix} \mathbf{x}^{(t)} \\ \mathbf{h}^{(t-1)} \end{pmatrix} \times_2 \cdots \times_p \begin{pmatrix} \mathbf{x}^{(t)} \\ \mathbf{h}^{(t-1)} \end{pmatrix}}_{p\text{-fold tensor product with itself}} = \mathcal{G} \cdot \begin{pmatrix} \mathbf{x}^{(t)} \\ \mathbf{h}^{(t-1)} \end{pmatrix}^{\otimes p}$$

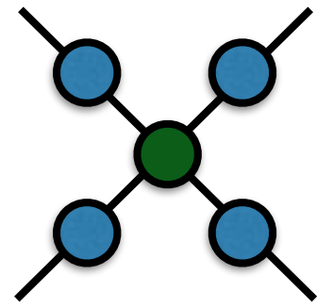
Large p leads to long memory, small p leads to short memory

Tensor Networks in Deep Learning

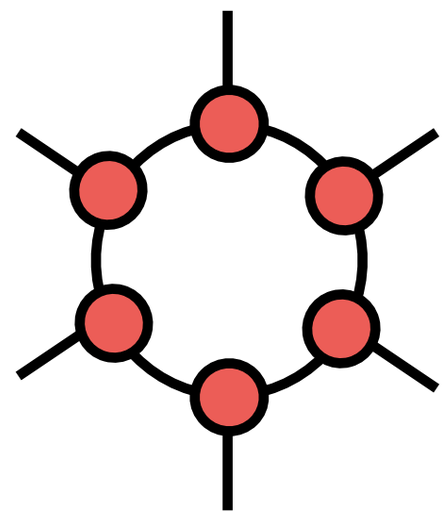
Full-connected network
(Novikov et al., 2015)

$$\mathcal{Y} = \langle \mathcal{W}, \phi(\mathcal{X}) \rangle =$$  $$, \phi(\mathcal{X})$$

Regression network
(Kossaifi et al., 2020)

$$\mathcal{Y} = \langle \mathcal{W}, \phi(\mathcal{X}) \rangle =$$  $$, \phi(\mathcal{X})$$

Convolutional network
(Wang et al., 2019)

$$\mathcal{Y} = \langle \mathcal{W}, \phi(\mathcal{X}) \rangle =$$  $$, \phi(\mathcal{X})$$

Which is the optimal TN structure for machine learning tasks?

Tensor Network Structure Search (TN-SS)

Involved Algorithm

Spectral Decomp.

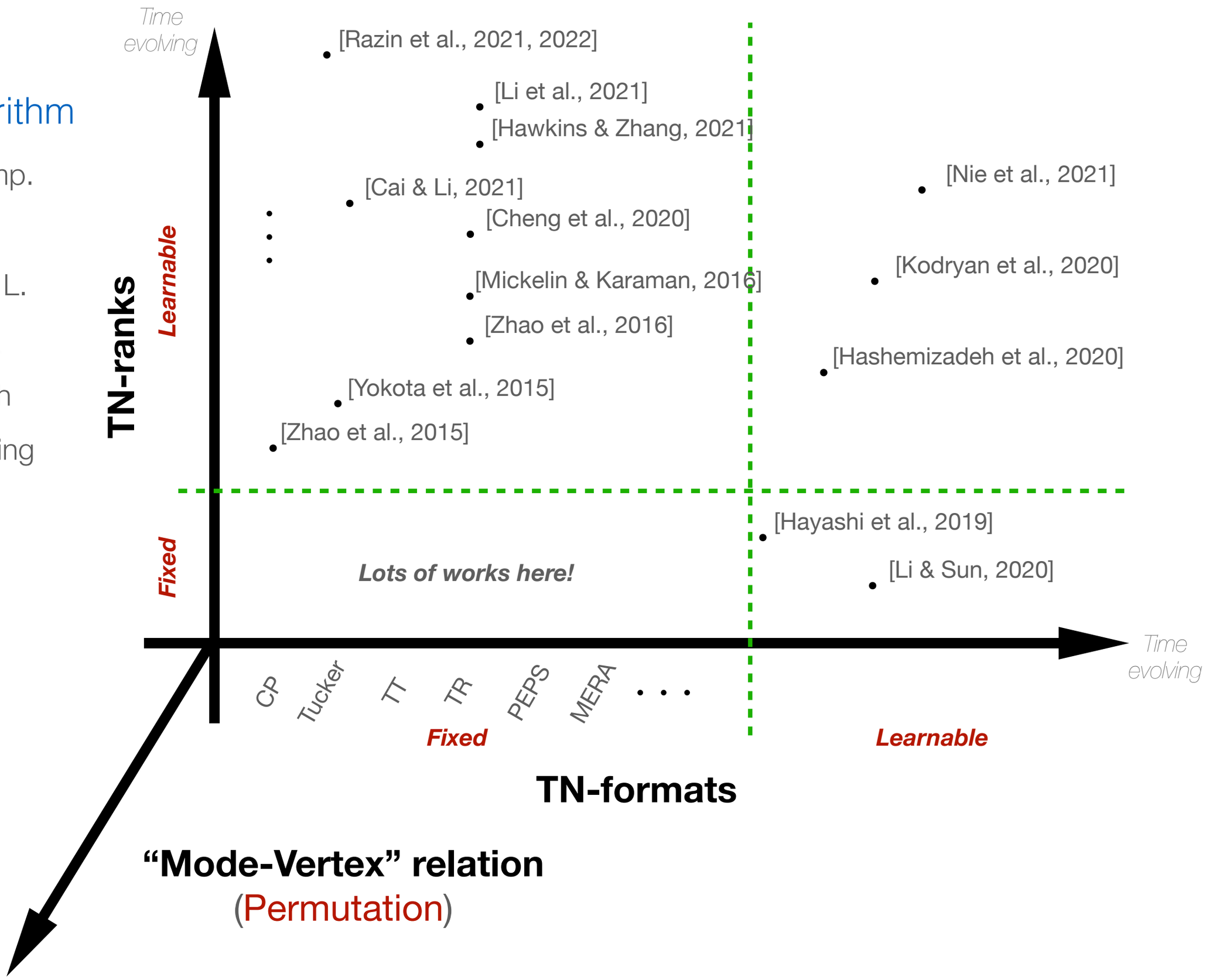
Bayesian Inf.

Reinforcement L.

Implicit regul.

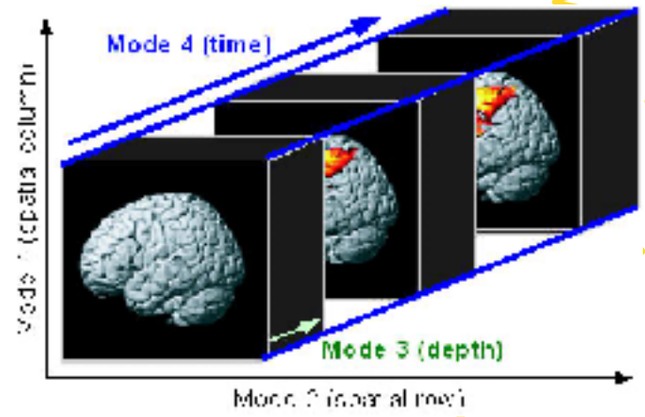
Greedy search

Random sampling



Tensor Network Permutation Search (TN-PS)

(Li et al., ICML 2021)

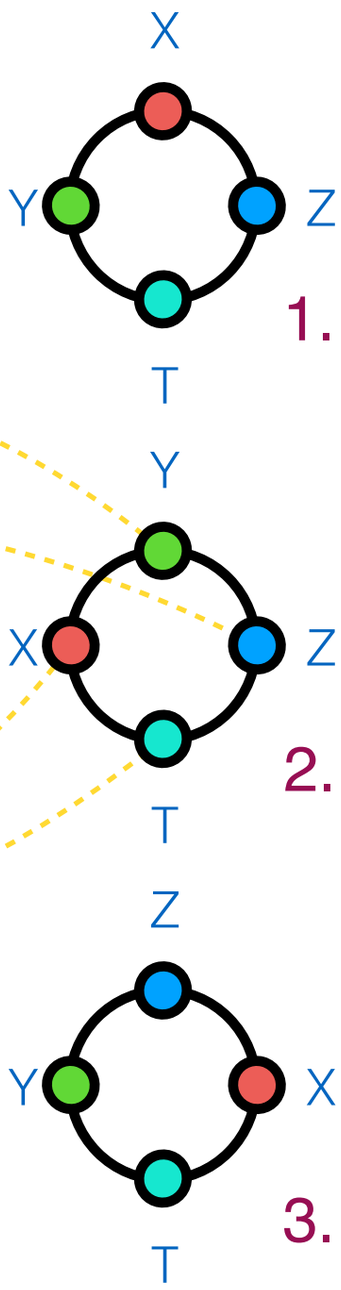


4th-order tensor

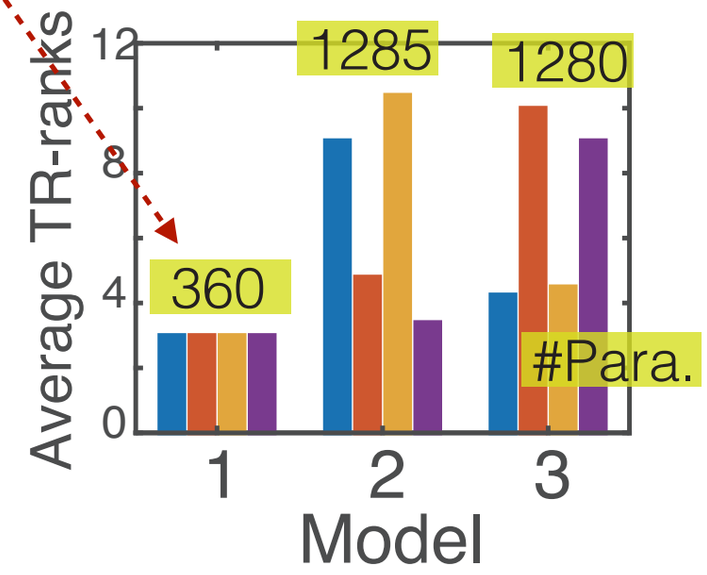
X,Y,Z-axis and T (time)

TN approx.

cycle



Fewer parameters



*TR approx. on synthetic data

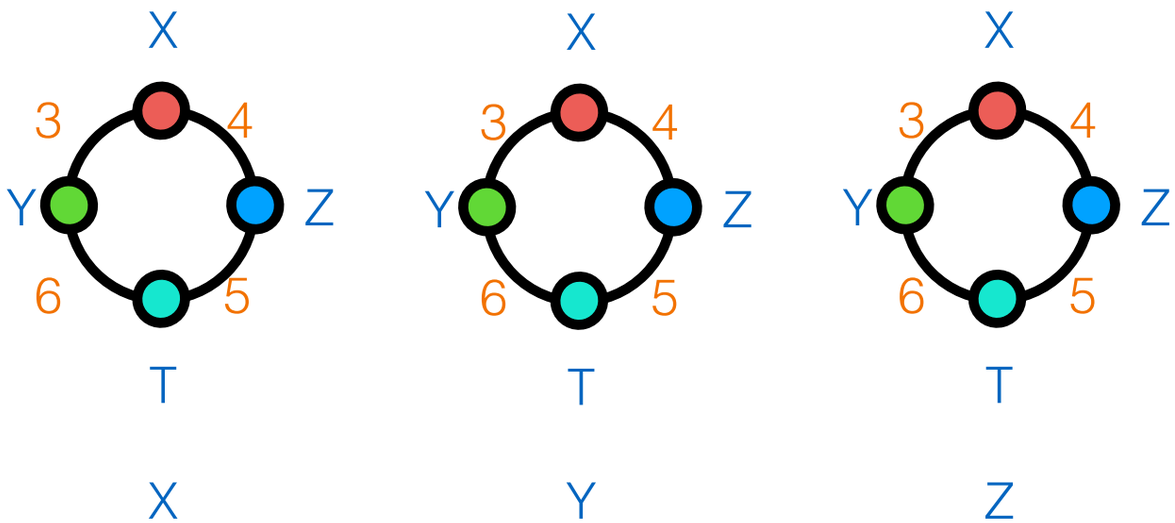
The goal of TN-PS is to search for good “mode-vertex” mappings and ranks.

Image source: https://staffwww.dcs.shef.ac.uk/people/H.Lu/feeler.html

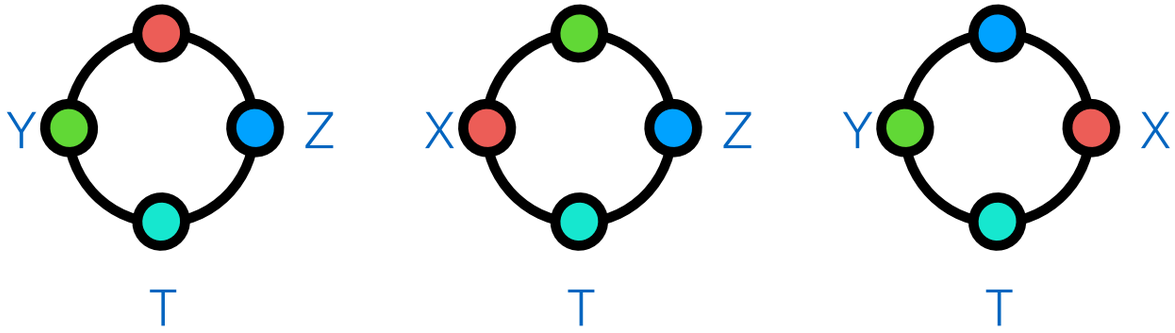
TN Structure Search

**The dangling edges are ignored.*

TN-RS
(Rank, edge labels)

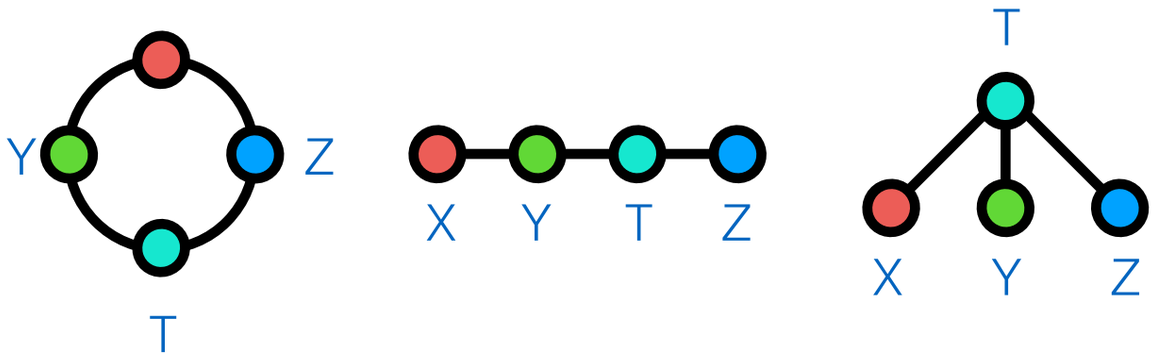


TN-PS
(Vertex Permutation)



(Li et al., ICML 2021)

TN-TS
(Network Topology)



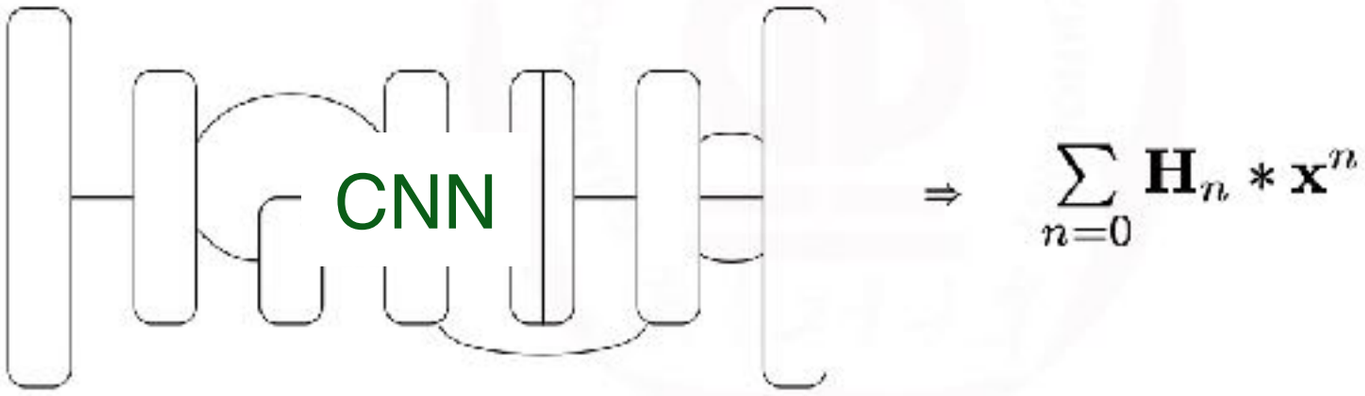
(Li et al., ICML 2020)

Image source: https://staffwww.dcs.shef.ac.uk/people/H.Lu/feeler.html

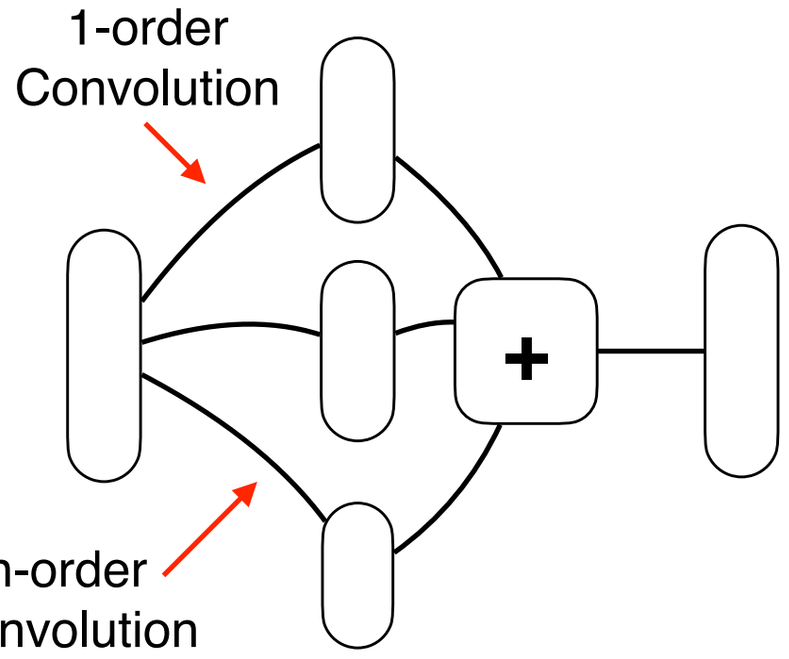
Understanding CNN from Volterra Convolution Perspective

(Li et al. JMLR 2022)

- **Theorem**: Most convolutional neural networks can be interpreted as a form of Volterra convolutions.



$$\Rightarrow \quad \sum_{n=0} \mathbf{H}_n * \mathbf{x}^n$$



- **Volterra Convolution**

n-order kernel tensor

$$\left(\sum_{n=0}^{+\infty} \mathbf{H}_n * \mathbf{x}^n\right)(t) = \sum_{n=0}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} H_n(\tau_1, \cdots, \tau_n) \prod_{i=1}^{n} (x(t-\tau_i) d\tau_i)$$

NOT n-dimensional convolution

Example

Lemma

The "conv | act | conv" structure can be converted to the form of Volterra convolution.

Taylor series expansion of activation function

- $\mathbf{g} * \sigma(\mathbf{h} * \mathbf{x})$

$$= \mathbf{g} * \left(\sigma(0) + \sigma'(0)[\mathbf{h} * \mathbf{x}] + \frac{\sigma''(0)}{2!}[\mathbf{h} * \mathbf{x}]^2 + \frac{\sigma'''(0)}{3!}[\mathbf{h} * \mathbf{x}]^3 + \cdots \right)$$

$$= \sigma(0) \sum \mathbf{g} + \sigma'(0)(\mathbf{g} \circledast \mathbf{h}) * \mathbf{x} + \frac{\sigma''(0)}{2!}(\mathrm{diag}(2, \mathbf{g}) \circledast \mathbf{h}^2) * \mathbf{x}^2 + \frac{\sigma'''(0)}{3!}(\mathrm{diag}(3, \mathbf{g}) \circledast \mathbf{h}^3) * \mathbf{x}^3 + \cdots .$$

Lemma

The "order-n | order-m" structure with $n, m > 0$ can be converted to the form of order-nm Volterra convolution.

$$\mathbf{y} = \sum_{i=0}^{n} \mathbf{H}_i * \mathbf{x}^i, \quad \mathbf{z} = \sum_{j=0}^{m} \mathbf{G}_j * \mathbf{y}^j$$

$$\Rightarrow \mathbf{z} = \sum_{j=0}^{m} \left(\sum \binom{j}{j_0 j_1 \cdots j_n} \left(\mathbf{G}_j \circledast \left[\mathbf{H}_0^{j_0}, \mathbf{H}_1^{j_1}, \cdots, \mathbf{H}_n^{j_n} \right] \right) * \mathbf{x}^{j_1 + 2j_2 + \cdots + nj_n} \right),$$

The order grows exponentially with number of layer

Black-box Attack by Volterra Convolution

(Li et al. JMLR 2022)

Well trained CNN → VC representation

- **Direct computation** (white box) or **training a VC network** by proxy kernels (black box)
- The perturbation computed by attacking VC can also **attack original CNN**.

- **Upper bound** w.r.t. perturbation

Theorem 19 *Assume input signal is* $\mathbf{x}$*, and the perturbation is* ϵ*, the approximated neural network is* $f(\mathbf{x}) = \sum_{n=0}^{N} \mathbf{H}_n * \mathbf{x}^n$*, we have*

$$\|f(\mathbf{x}+\epsilon) - f(\mathbf{x})\|_2 \leq \min \begin{pmatrix} \sum_{n=0}^{N} \|\mathbf{H}_n\|_2 \sum_{k=0}^{n-1} \left(\frac{en}{k}\right)^k \|\mathbf{x}\|_1^k \|\epsilon\|_1^{n-k}, \\ \sum_{n=0}^{N} \|\mathbf{H}_n\|_1 \sum_{k=0}^{n-1} \left(\frac{en}{k}\right)^k \|\mathbf{x}\|_{2k}^k \|c\|_{2(n-k)}^{n-k} \end{pmatrix}, \tag{34}$$

where $e = 2.718281828\cdots$*, the base of the natural logarithm.*

Computational Efficiency

Discovering efficient algorithms in mathematics

- Matrix multiplication: ubiquitous in NNs and modern computing
  - Developing computing hardware (large amounts of time and money)
  - Finding the fastest algorithm (50-year-old open question, difficult problem in mathematics)

- Example: 2 x 2 matrices

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \times \begin{bmatrix} b_{1,1} & b_{1,2} \\ b_{2,1} & b_{2,2} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{bmatrix}$$

- Unsolved problem in larger matrix cases
- Automatic algorithm discovery by AI

[Fawzi et al. Nature 2022]

Standard algorithm

$h_1 = a_{1,1}\, b_{1,1}$
$h_2 = a_{1,2}\, b_{2,1}$
$h_3 = a_{1,1}\, b_{1,2}$
$h_4 = a_{1,2}\, b_{2,2}$
$h_5 = a_{2,1}\, b_{1,1}$
$h_6 = a_{2,1}\, b_{1,2}$
$h_7 = a_{2,2}\, b_{2,1}$
$h_8 = a_{2,2}\, b_{2,2}$

$c_{1,1} = h_1 + h_2$
$c_{1,2} = h_3 + h_4$
$c_{2,1} = h_5 + h_7$
$c_{2,2} = h_6 + h_8$

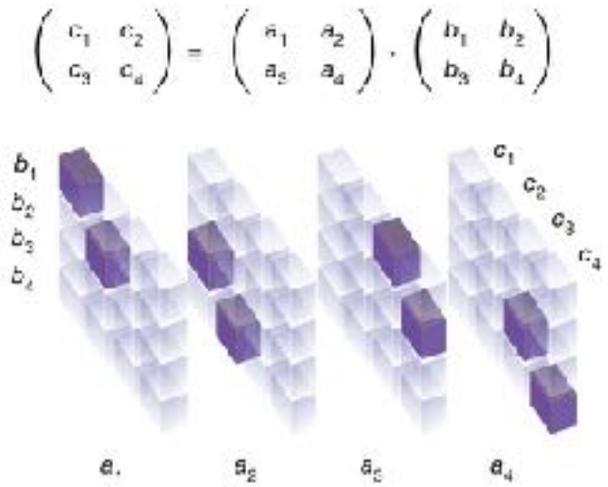
Strassen's algorithm

$h_1 = (a_{1,1} + a_{2,2})\,(b_{1,1} + b_{2,2})$
$h_2 = (a_{2,1} + a_{2,2})\, b_{1,1}$
$h_3 = a_{1,1}\,(b_{1,2} - b_{2,2})$
$h_4 = a_{2,2}\,(-b_{1,1} + b_{2,1})$
$h_5 = (a_{1,1} + a_{1,2})\, b_{2,2}$
$h_6 = (-a_{1,1} + a_{2,1})\,(b_{1,1} + b_{1,2})$
$h_7 = (a_{1,2} - a_{2,2})(b_{2,1} + b_{2,2})$

$c_{1,1} = h_1 + h_4 - h_5 + h_7$
$c_{1,2} = h_3 + h_5$
$c_{2,1} = h_2 + h_4$
$c_{2,2} = h_1 - h_2 + h_3 + h_6$

AlphaTensor: Discovering novel algorithms using Tensor Decomposition

Encoding



Tensor Decomposition

$$\mathbf{U} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{W} = \begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathcal{T}_n = \sum_{r=1}^{R} \mathbf{u}^{(r)} \otimes \mathbf{v}^{(r)} \otimes \mathbf{w}^{(r)},$$

Decoding

$$m_1 = (a_1 + a_4)(b_1 + b_4)$$
$$m_2 = (a_3 + a_4)\, b_1$$
$$m_3 = a_1\,(b_2 - b_4)$$
$$m_4 = a_4\,(b_3 - b_1)$$
$$m_5 = (a_1 + a_2)\, b_4$$
$$m_6 = (a_3 - a_1)(b_1 + b_2)$$
$$m_7 = (a_2 - a_4)(b_3 + b_4)$$
$$c_1 = m_1 + m_4 - m_5 + m_7$$
$$c_2 = m_3 + m_5$$
$$c_3 = m_2 + m_4$$
$$c_4 = m_1 - m_2 + m_3 + m_6$$

Rank of CPD determines the minimum number of multiplications

[Fawzi et al. Nature 2022]

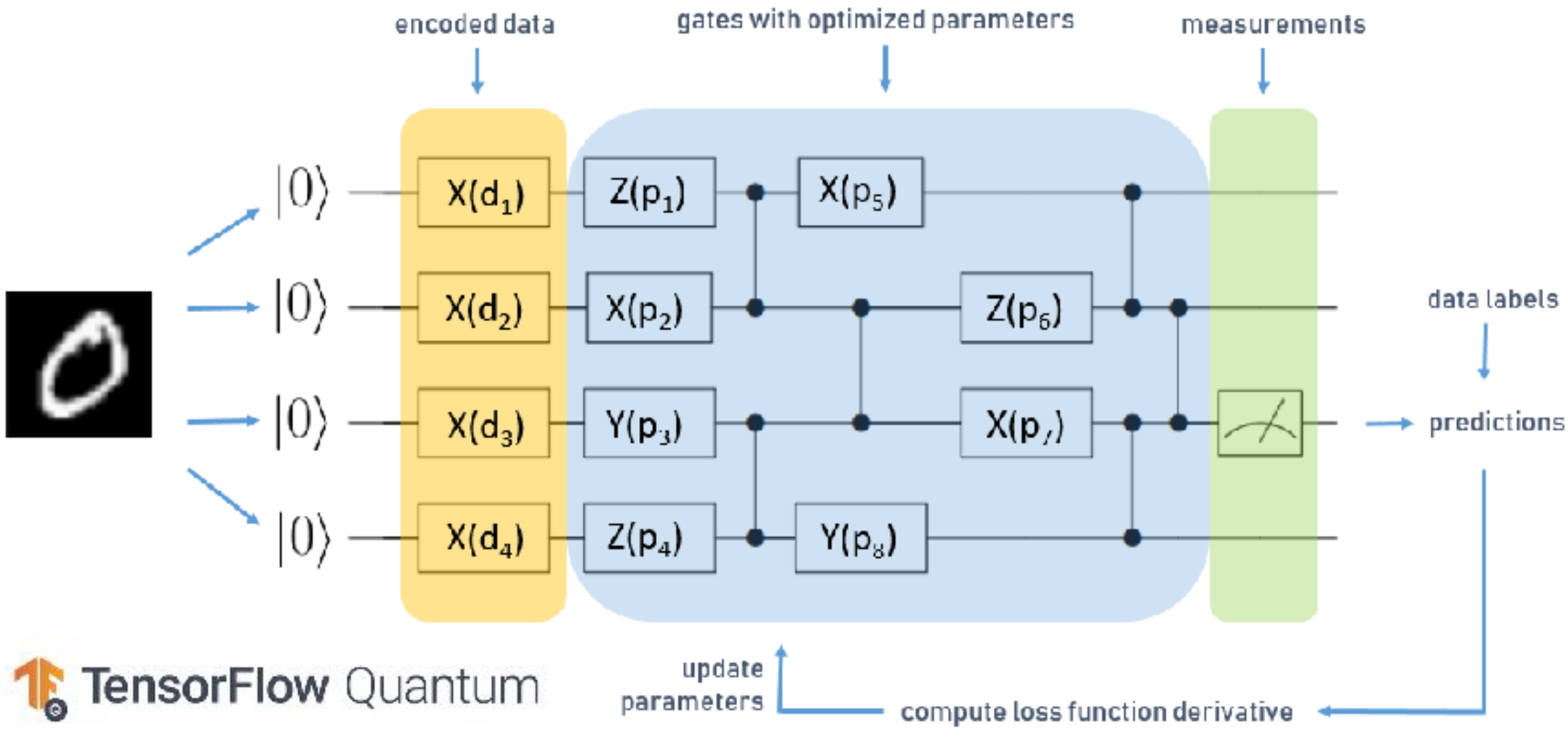
AlphaTensor: Discovering novel algorithms in mathematics

Size (n, m, p)	Best method known	Best rank known	AlphaTensor rank Modular	AlphaTensor rank Standard
(2, 2, 2)	(Strassen, 1969)[2]	7	7	7
(3, 3, 3)	(Laderman, 1976)[15]	23	23	23
(4, 4, 4)	(Strassen, 1969)[2] $(2, 2, 2) \otimes (2, 2, 2)$	49	47	49
(5, 5, 5)	(3, 5, 5) + (2, 5, 5)	98	96	98

- Discovered algorithm outperforms the two-level Strassen's algorithm (best human knowledge).
- One week later, *Manuel Kauers* and *Jakob Moosbauer* beat AlphaTensor (5 x 5 matrix , 96 -> 95). [Kauers et al. ArXiv 2022]

[Fawzi et al. Nature 2022]

Quantum Machine Learning



- Limited qubits with small scale data and model.
- Performance on ML tasks cannot compete with classical ML.

https://blog.tensorflow.org/2020/08/layerwise-learning-for-quantum-neural-networks.html

Summary

- TNs are powerful tools for representation of high-dimensional structured data.
- TNs are efficient reparameterization of deep learning models.
- However, there are some problems need to further solved prior to the real-world applications, such as TN-SS.
- Robustness to adversarial attacks, and interpretability of TN based models.
- Quantum machine learning is an potentially promising technology.

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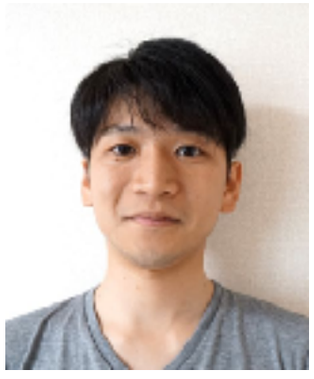

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