Simulating quantum computers with probabilistic methods: Tokyo ImPACT lecture 2 – quantum emulation

P. D. Drummond

SWINBURNE UNIVERSITY OF TECHNOLOGY

Quantum science: Nobel award, 2012

- Background: **Paradoxes in quantum mechanics?**
  Measurement,   EPR paradoxes
- Wineland: **Schrodinger's Cat with ions**
  Ion traps, quantum logic, atomic clocks
- Haroche: **Schrodinger Cat with photons**
  Microwave resonators, Quantum circuits
- New technologies: **where is this heading?**
  Quantum cryptography - secure communications
  Quantum computing and emulation
  Gravity wave detectors and nano-mechanics
  Ultra-cold atoms and atom lasers

21^{st} century QC development

- 20^{th} century: universal QC < 10 qubit

– growth < 1qubit/yr ☹, for factorization need $>10^6$

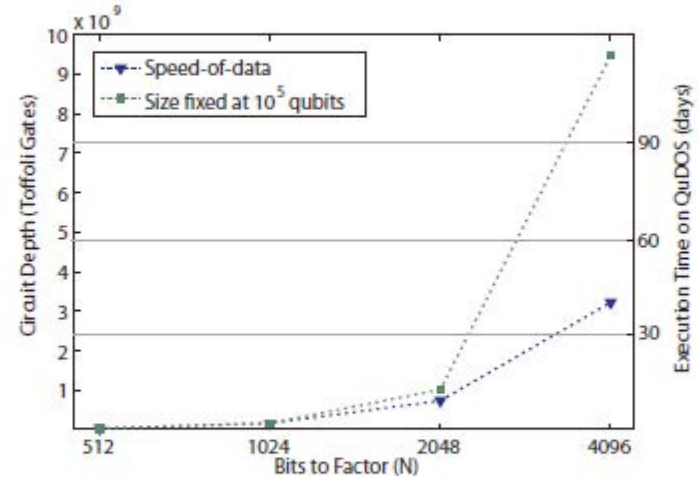
- 21^{st} century: targeted QC, NP & #P hard problems

– D-wave QC, Google, NASA;`Ising' QC , Stanford.

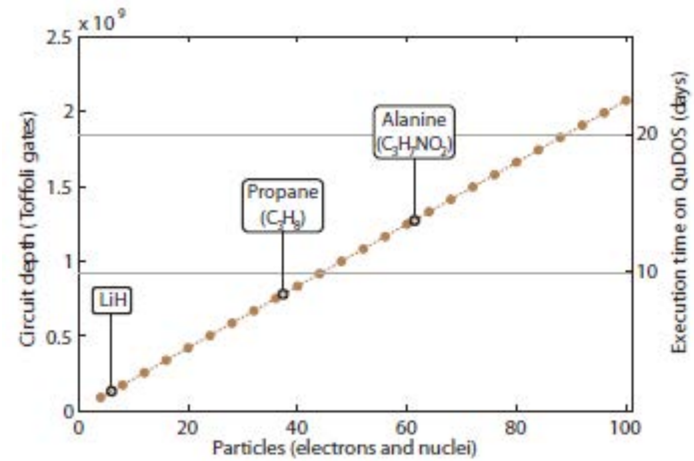
- This talk : new technologies and proposals
    - **QFT dynamics: early universe BEC simulator**
    - **Boson sampling: #P-hard problems, metrology**
    - **Coupled paramps: 'Light meets magnetism'**

Resources for Fault-Tolerant Q Computer

Computing Resource	Factoring (1024-bit)	Molecular Simulation (alanine)
Application qubits	6144	6649
Distillation qubits	66564	15860
Code distance	31	33
Error rate per lattice cycle	6.53×10^{-19}	5.94×10^{-20}
Error rate per virtual gate	10^{-3}	10^{-3}
Physical qubits	4.54×10^{8}	1.73×10^{8}
Circuit depth (Toffoli gate)	1.68×10^{8}	1.27×10^{9}
Execution time	1.81 days	13.7 days

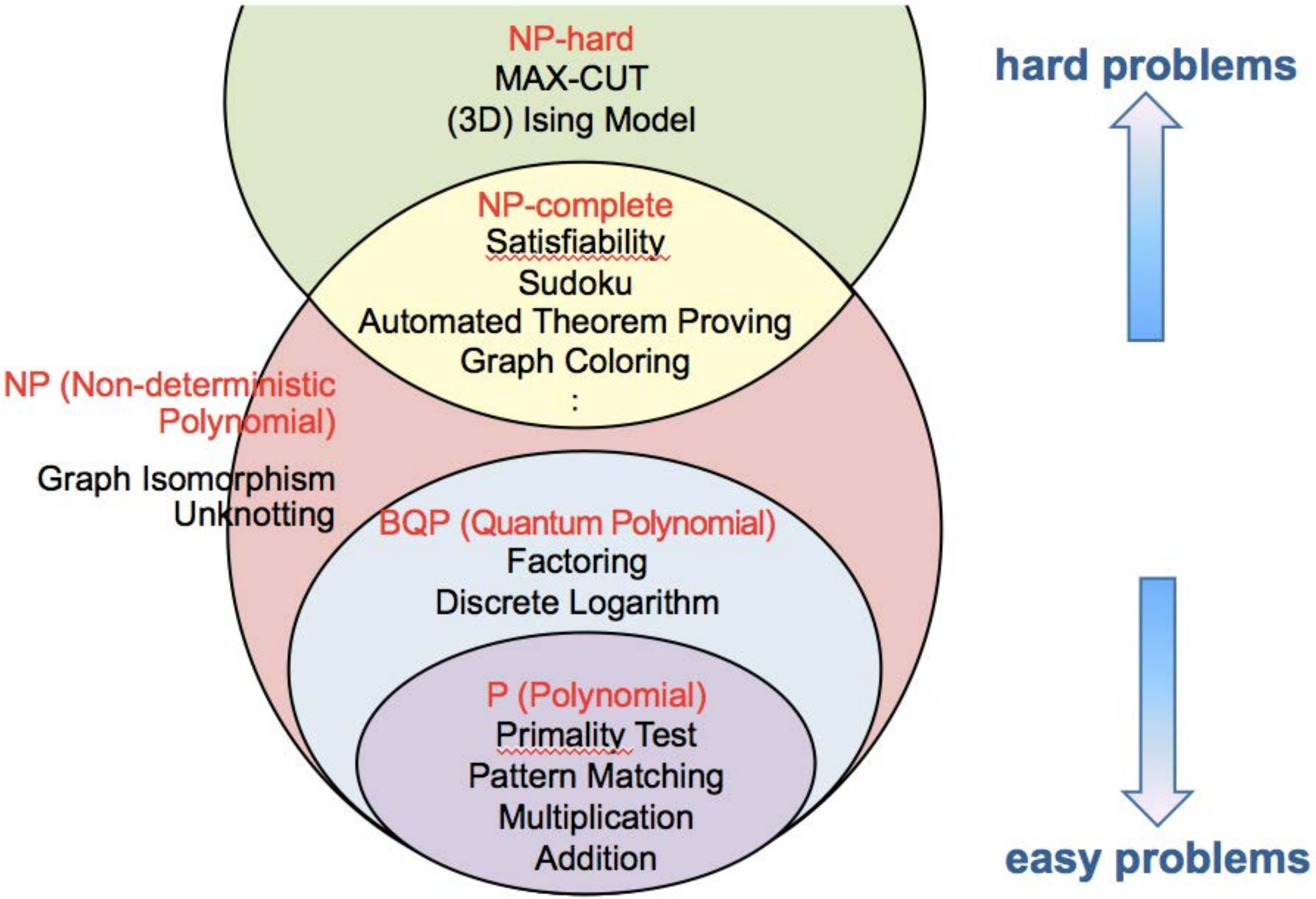


N.C. Jones et. al.,
Phys. Rev. X2, 031007 (2012)



N.C. Jones et. al.,
New J. Phys. 14, 115023 (2012)

Moving up in the complexity hierarchy



Quantum computing technologies

	Qu. Computing	Qu. Annealing	Qu. Ising
Physics	SU(2) rotation	adiabatic evolution	dissipative evolution
Information	spin-1/2 particles	spin-1/2 particles	photonic (non-local)
Principle	interferometer	tunneling landscape	SSB at critical point
Dissipation	need QEC	needs QEC(?)	use as resource
Temp.	cryogenic (10mK)	cryogenic (10mK)	room temperature
Application	factoring (BQP)	optimization (NP)	optimization (NP)

Developing technologies

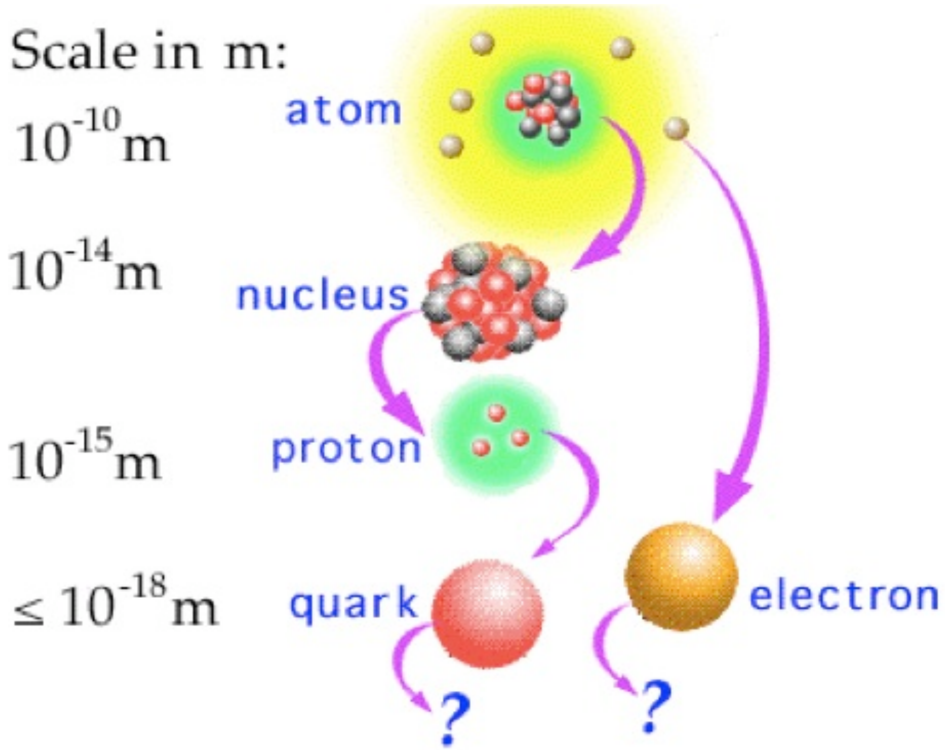
	Qu. Emulation	Qu. Boson Sampling	Qu. Paramp
Physics	field dynamics	SU(n) rotation	dissipative evolution
Information	atomic fields	photons	polaritons
Principle	H engineering	high order correlations	Lifshitz point
Dissipation	needs control	reduces counts	use as resource
Temp.	cryogenic (10nK)	room temperature	room temperature
Application	QFT, cosmology	BosonSampling (#P)	?

Quantum emulation

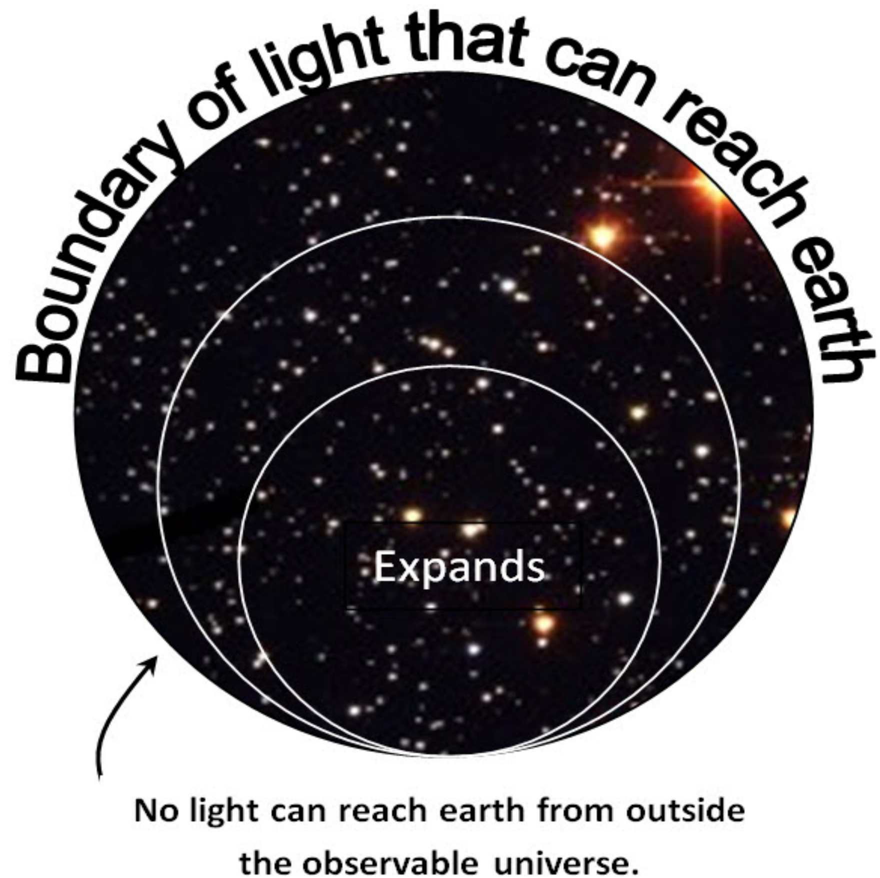
- Quantum field theory: exponentially complex
  - Essential to current theories of cosmology
  - Energies a trillion times larger than CERN
  - How can we compute what theory predicts?
- **Use ultracold BEC as relativistic simulator**
  - Check predictions with approximate methods

The physically accessible Universe is finite

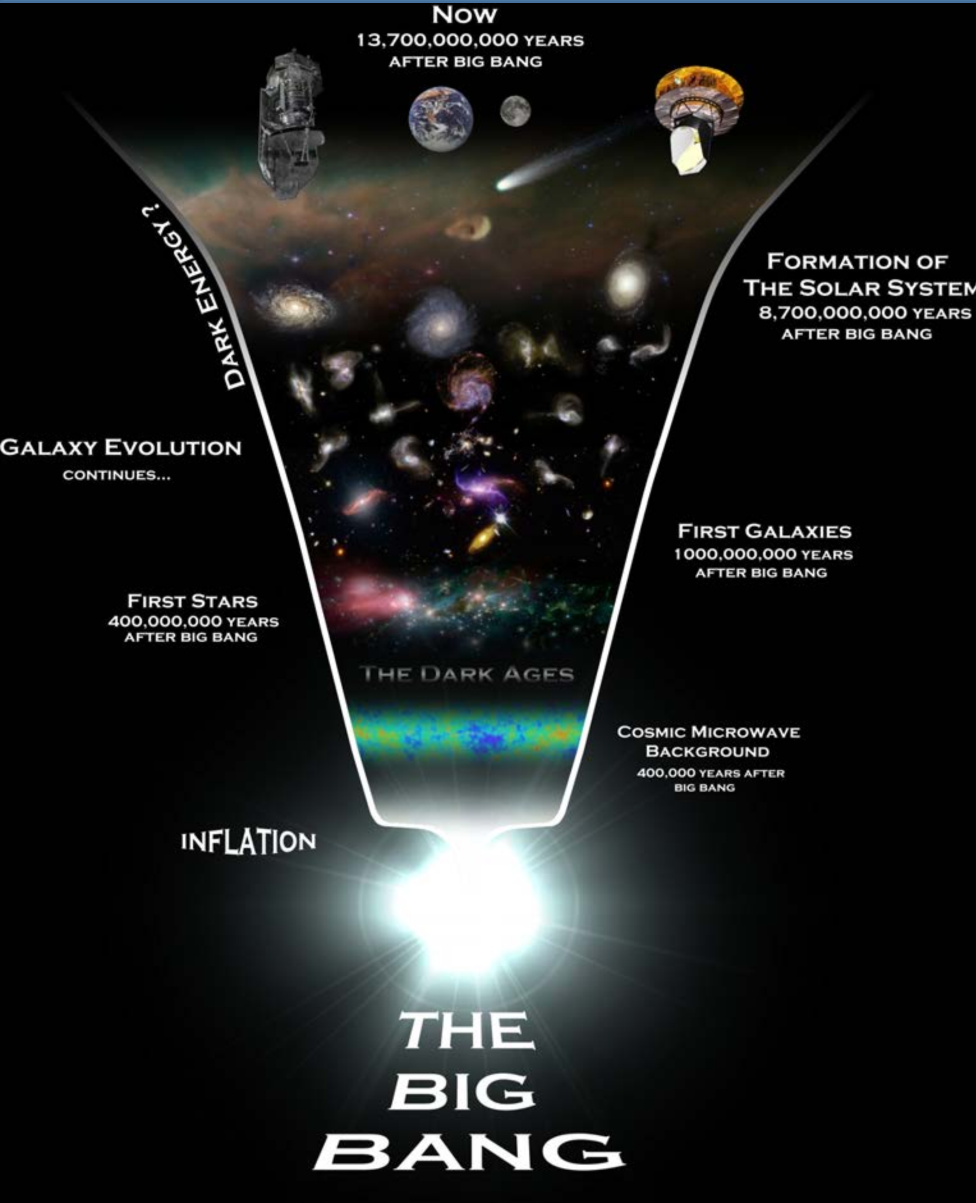
On the one hand, we are reaching the very small



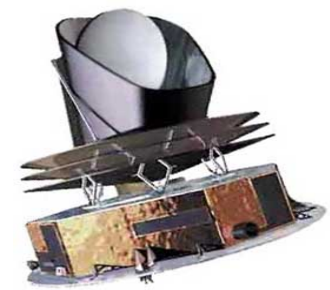
On the other hand, we can't see beyond the very large



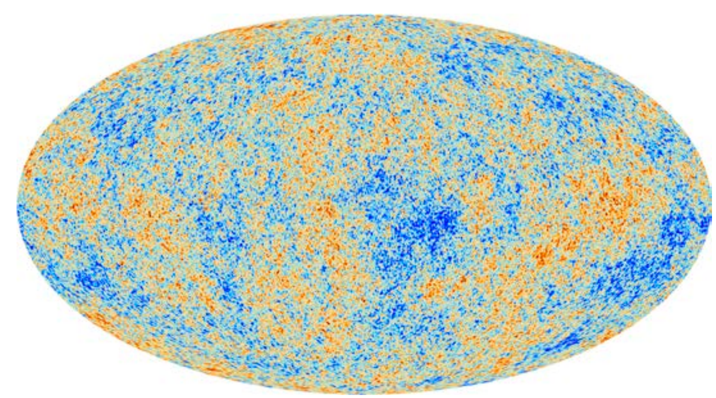
How can we test theories of the Big Bang?



Planck spacecraft was launched in May 2009. On 21 March 2013, the mission's all-sky map of the CMB was released



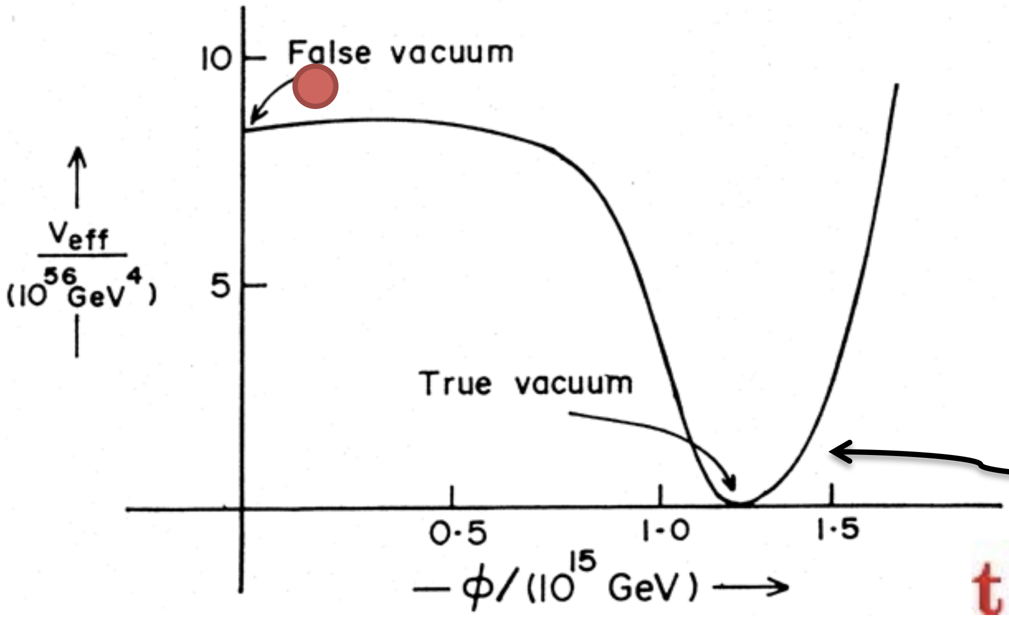
The CMB is a snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was just 380,000 years old.



We can't see beyond that

BGV theorem

Quantum models of the Big Bang



Potential energy on top of the hill is converted into kinetic energy of the rolling ball at the bottom of the hill.

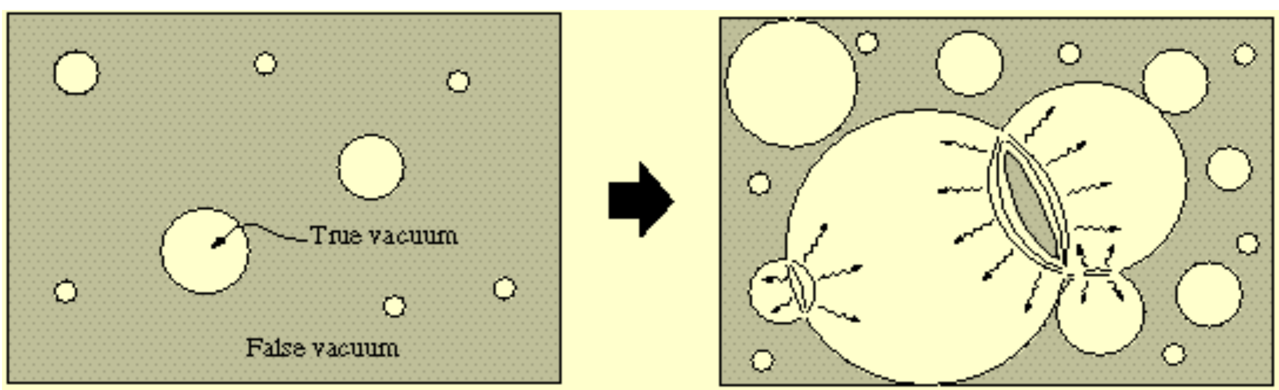
As a result a lot of energy was realised resulted in

BIG BANG

$t \sim 10^{-32}\,sec$

In reality the Universe has at least 3 dimensions. Bubbles appear during the transition to true vacuum.

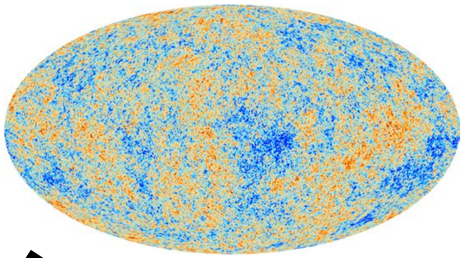
Are we in one of the bubbles….lonely….?

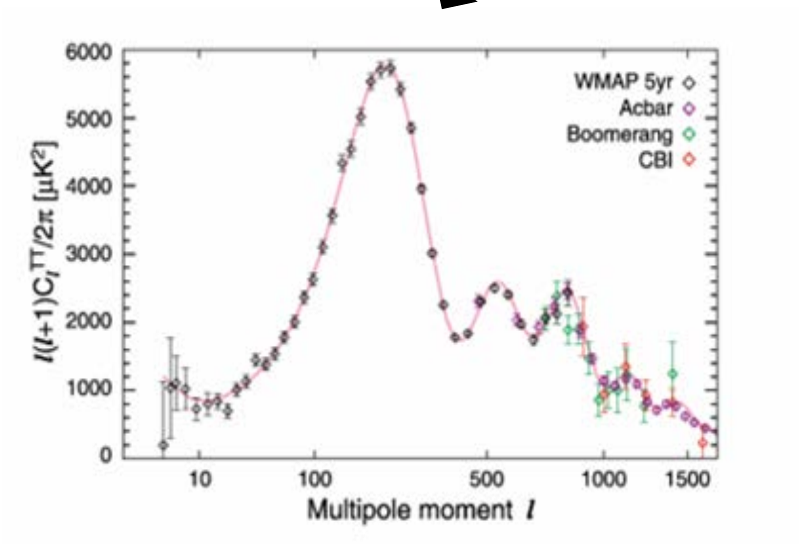


Similar to water boiling or bubbles in champagne

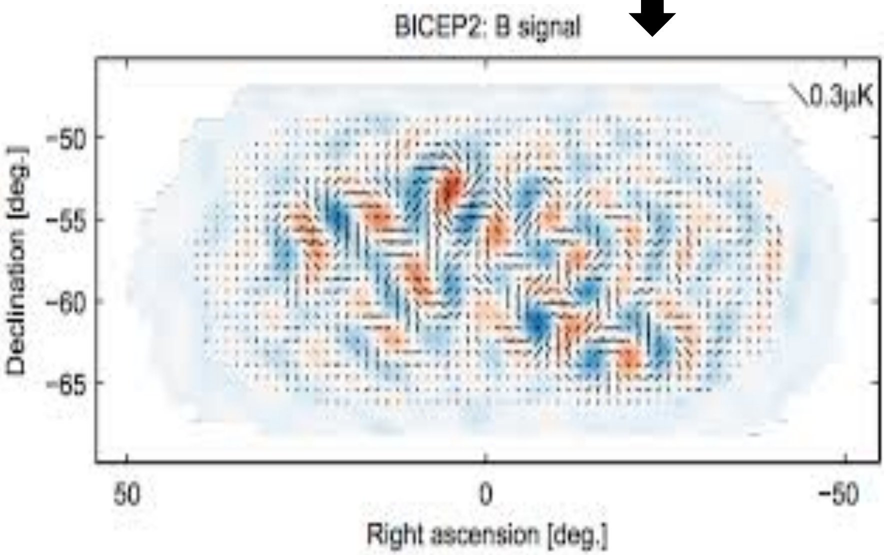
What is the observational evidence?

Planck spacecraft,
21 March 2013



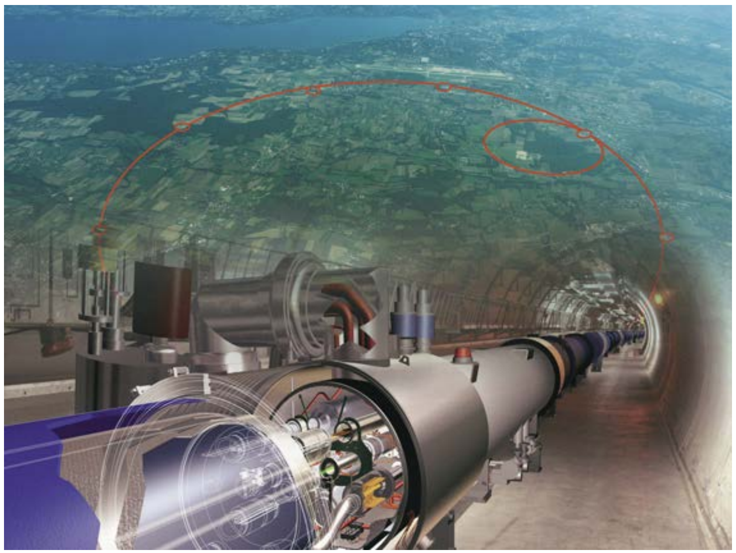


Bicep 2, Polar Bear,
South Pole Telescope



How can we test these models ?

LHC ? (completed in 2008, Higgs particle discovered in 2013, Nobel price 2013)

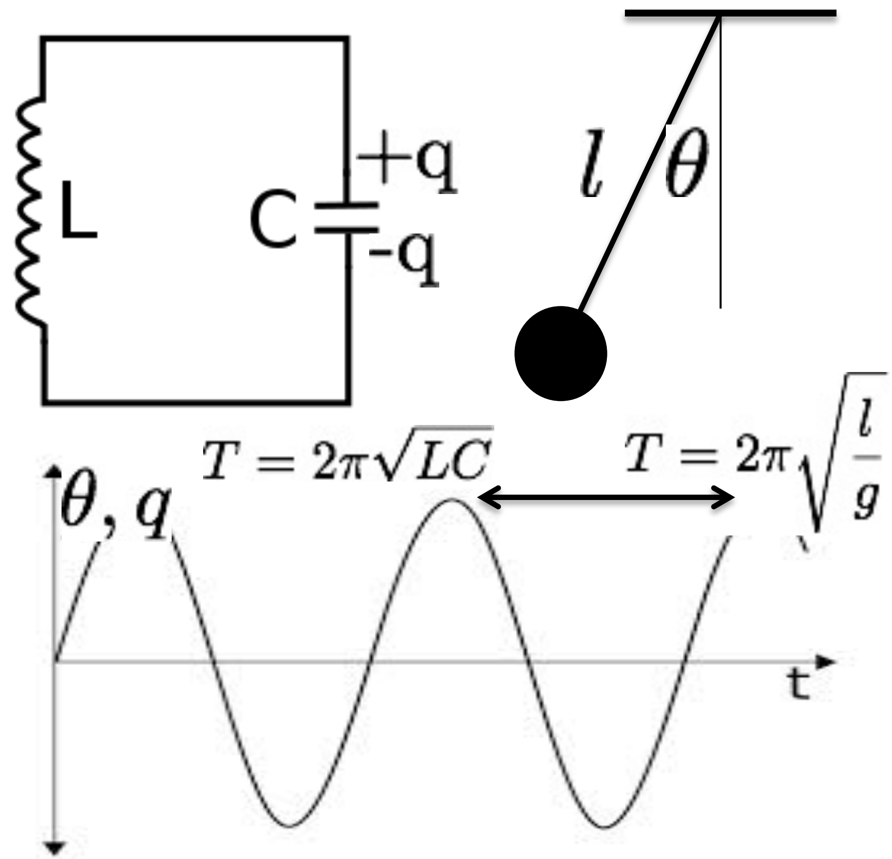


LHC is the world's largest and most powerful particle collider.

The energy density is 13 orders of magnitudes smaller than the energy density during cosmological inflation era !

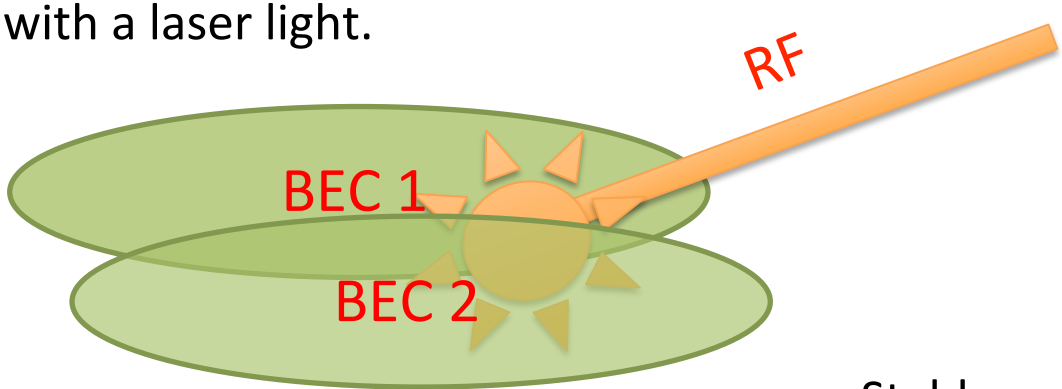
Use instead some analog physical system, which simulates the false vacuum decay

El. circuit $\sim$ pendulum

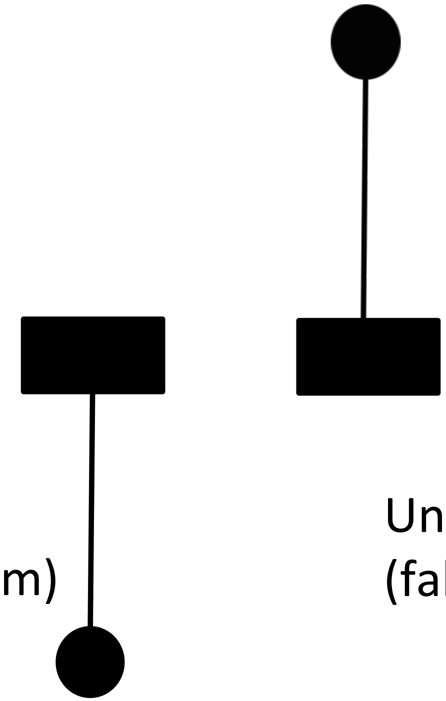


Analog quantum simulator

Take 2 BECs and couple them with a laser light.



$$\phi_1 - \phi_2 \approx \theta$$



Stable (true vacuum)

$\theta = 0$

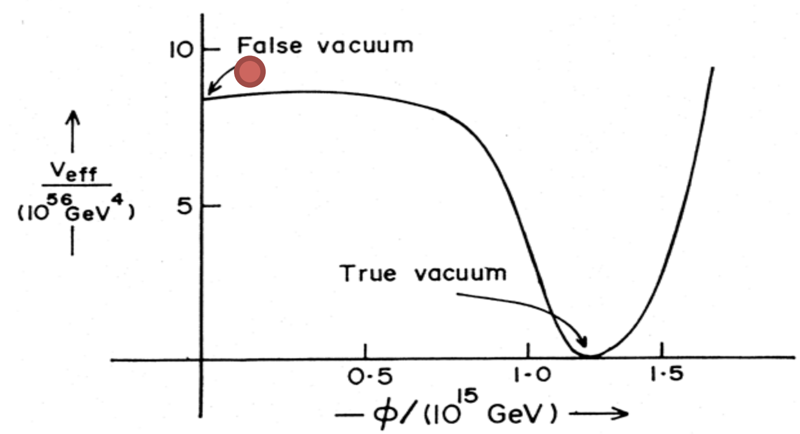
Unstable (false vacuum)

$\theta = \pi$

Their phase difference behaves like a pendulum, which has stable and unstable points.

$$\partial_t^2 \theta - c^2 \nabla^2 \theta = -\partial_\theta V(\theta)$$

-- relativistic field equation of the early Universe.



Early universe models

- The simplest model has a scalar inflaton field
- Relativistic, interacting quantum field dynamics
- $\phi(x)$ is described by the Lagrangian

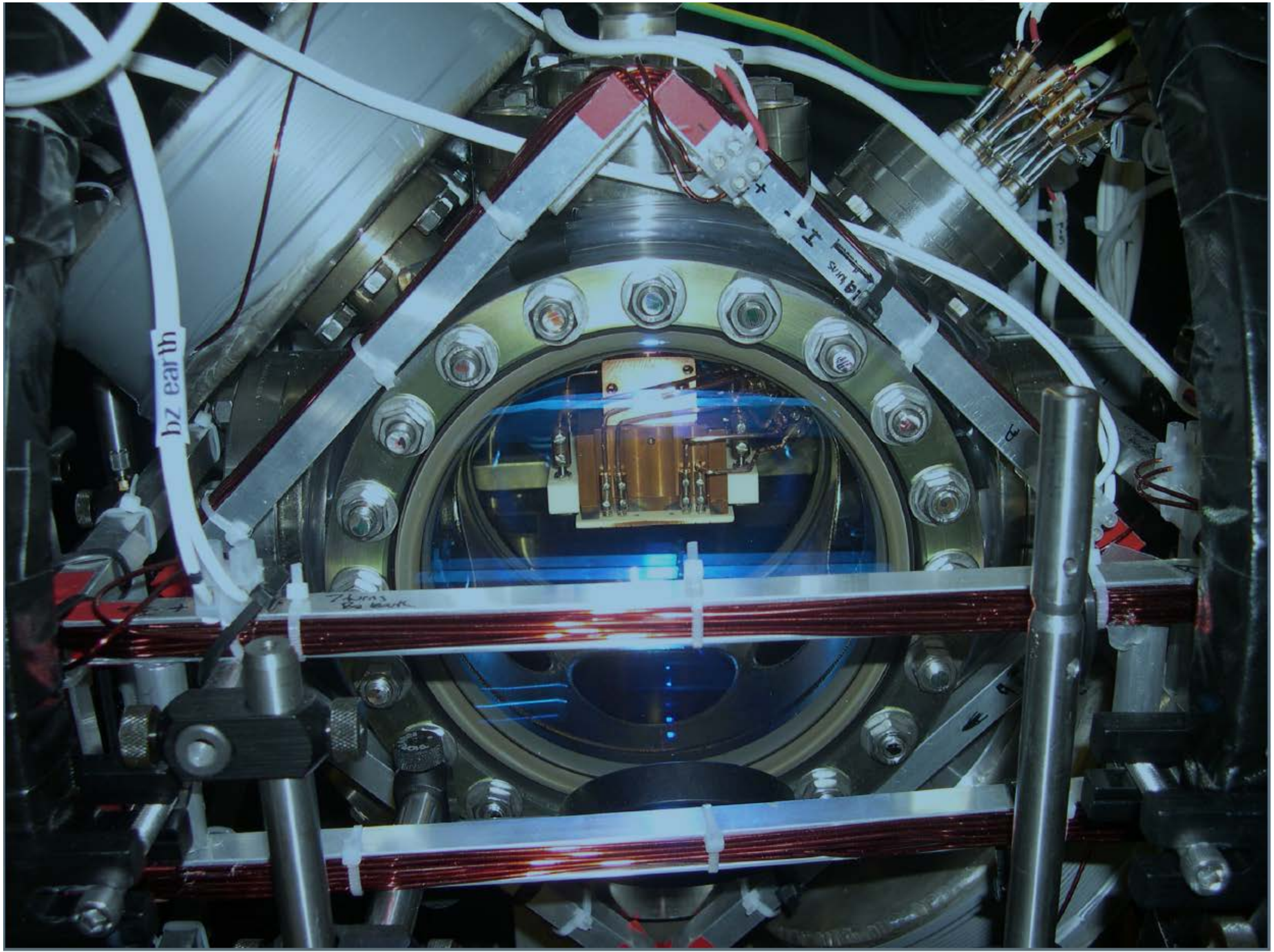
$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi),$$

where $V(\phi)$ is the potential down which the scalar field rolls

More complex models

- Need to assume a comoving, expanding frame
- Can include a background gravitational tensor
- Most inflation models have friction terms as well
- Also possible to have vector early universe fields
- First step: understand the simplest case

Test case: Interferometry on an atom chip (Sidorov, Swinburne)



Coupled Bose gas model master equation

A D-dimensional Bose gas has two spin components that are linearly coupled by an external microwave field.

$$\hat{H} = \hbar \int d^3\mathbf{x} \left[\frac{\hbar}{2m} \nabla \hat{\Psi}_i^\dagger \nabla \hat{\Psi}_i + V_i(\mathbf{x}) \hat{\Psi}_i^\dagger \hat{\Psi}_i + \frac{g_{ij}}{2} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i + \nu \hat{\Psi}_i^\dagger \hat{\Psi}_{3-i} \right]$$

Here, g_{ij} is the self- and cross-coupling in $D-$dimensions. Collisional damping follows a master equation,

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar} \left[\hat{H}, \hat{\rho} \right] + \sum \kappa_\ell \int d^3\mathbf{x} \left[2\hat{O}_\ell \hat{\rho} \hat{O}_\ell^\dagger - \hat{O}_\ell^\dagger \hat{O}_\ell \hat{\rho} - \hat{\rho} \hat{O}_\ell^\dagger \hat{O}_\ell \right]$$

This includes self- and cross nonlinear damping, with

$$\hat{O}_\ell = \prod \hat{\Psi}_j^{\ell_j}$$

Stochastic time-evolution equations

Result of Wigner operator mappings:

$$i\partial_\tau \psi_i = \left\{ -\frac{1}{2}\nabla_\zeta{}^2 + \gamma\psi_i^\dagger\psi_i + \gamma_c\psi_j^\dagger\psi_j \right\}\psi_i - \tilde{\nu}\psi_j,$$

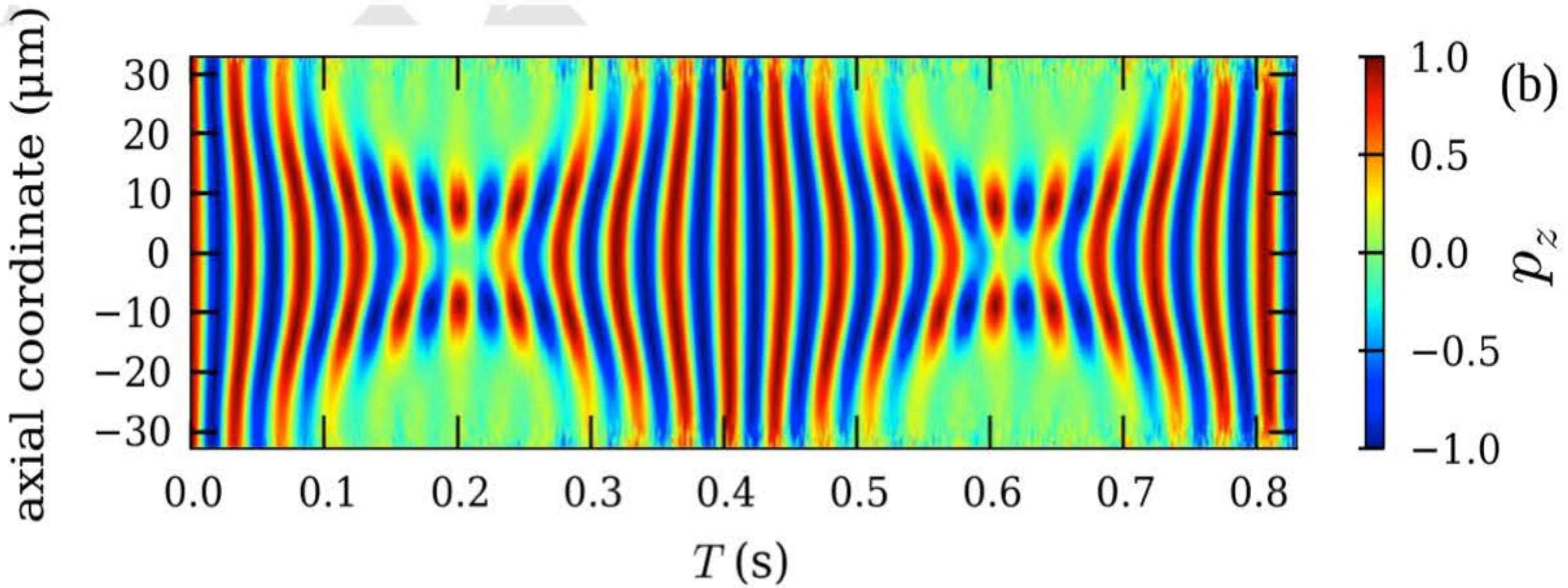
$$-\sum \tilde{\kappa}_\ell \frac{\partial \tilde{O}_\ell^*}{\partial \psi_i^*}\tilde{O}_\ell + B_{ij}[\boldsymbol{\psi}]\eta_j(t,x)$$

Scaling: $\tau = t/t_0$, $\zeta = x/x_0$,

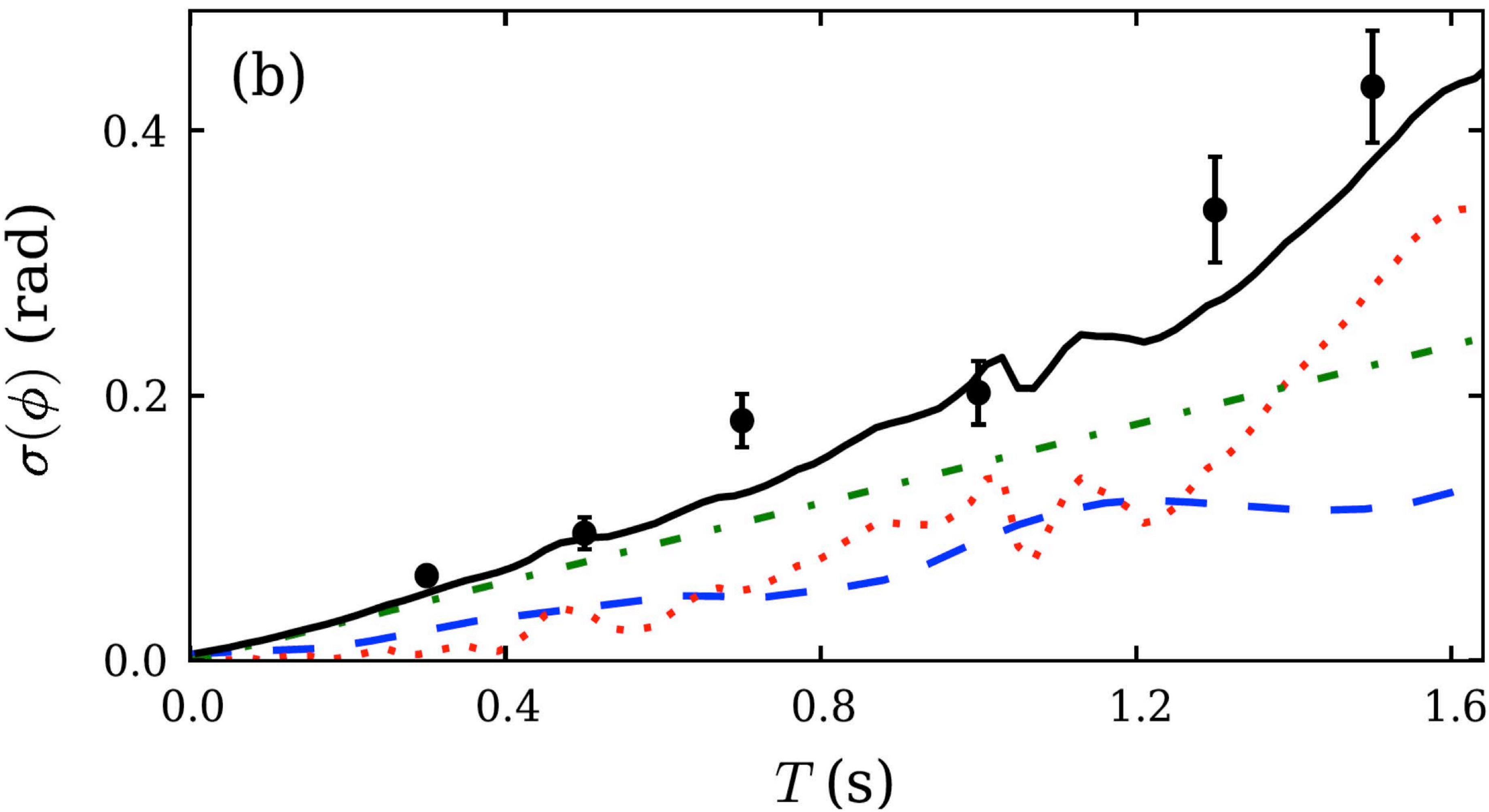
$t_0 = \hbar/gn$; $x_0 = \hbar/\sqrt{gnm}$; $\langle \Delta\tilde{\psi}(\zeta)\Delta\tilde{\psi}^*(\zeta')\rangle = \frac{1}{2}\delta(\zeta - \zeta')$.

Comparison to Rubidium experiment

A two-component, 4×10^4 atom ^{87}Rb BEC is in a harmonic trap with internal Zeeman states $|1, -1\rangle$ and $|2, 1\rangle$, which can be coupled via an RF field.



Wigner simulations vs phase noise

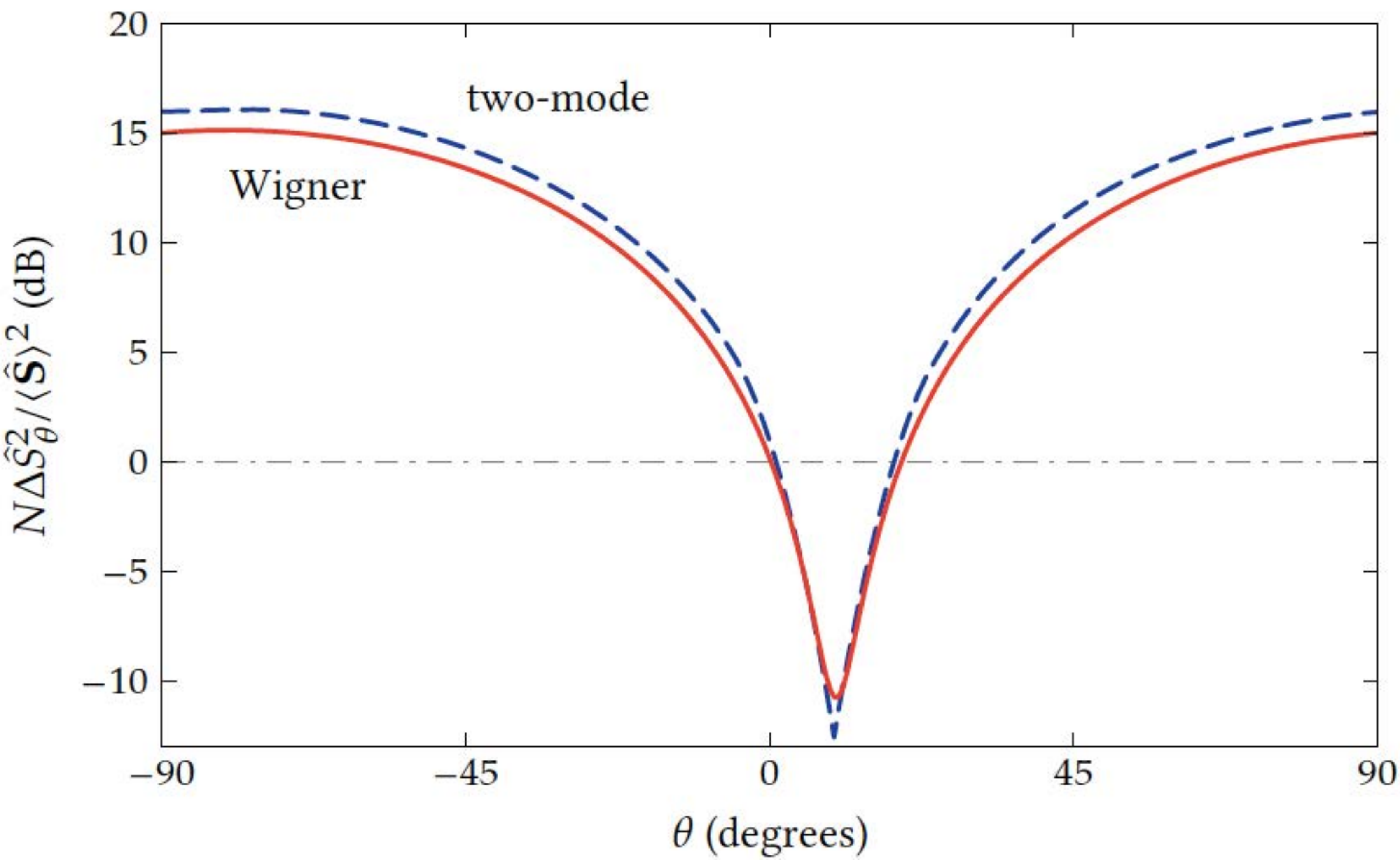


New SUT experiments underway

Previous experiments limited by:

- Uncertainties in initial atom number ~ 40,000
- Magnetic field fluctuations (unshielded)
- Microwave oscillator: only stable to one part in a trillion
- New experiments underway:
  - **six times less phase noise**

Wigner simulations of squeezing



Simulation of Treutlein experiment

Comparison: 1200 atom - experiment:

- Variational method (Sinatra) predicts -12.5dB
- Wigner method predicts -11dB squeezing
- Experiment observes -4.7dB
- Difference due to: technical noise? initial thermal noise?
- New simulations planned

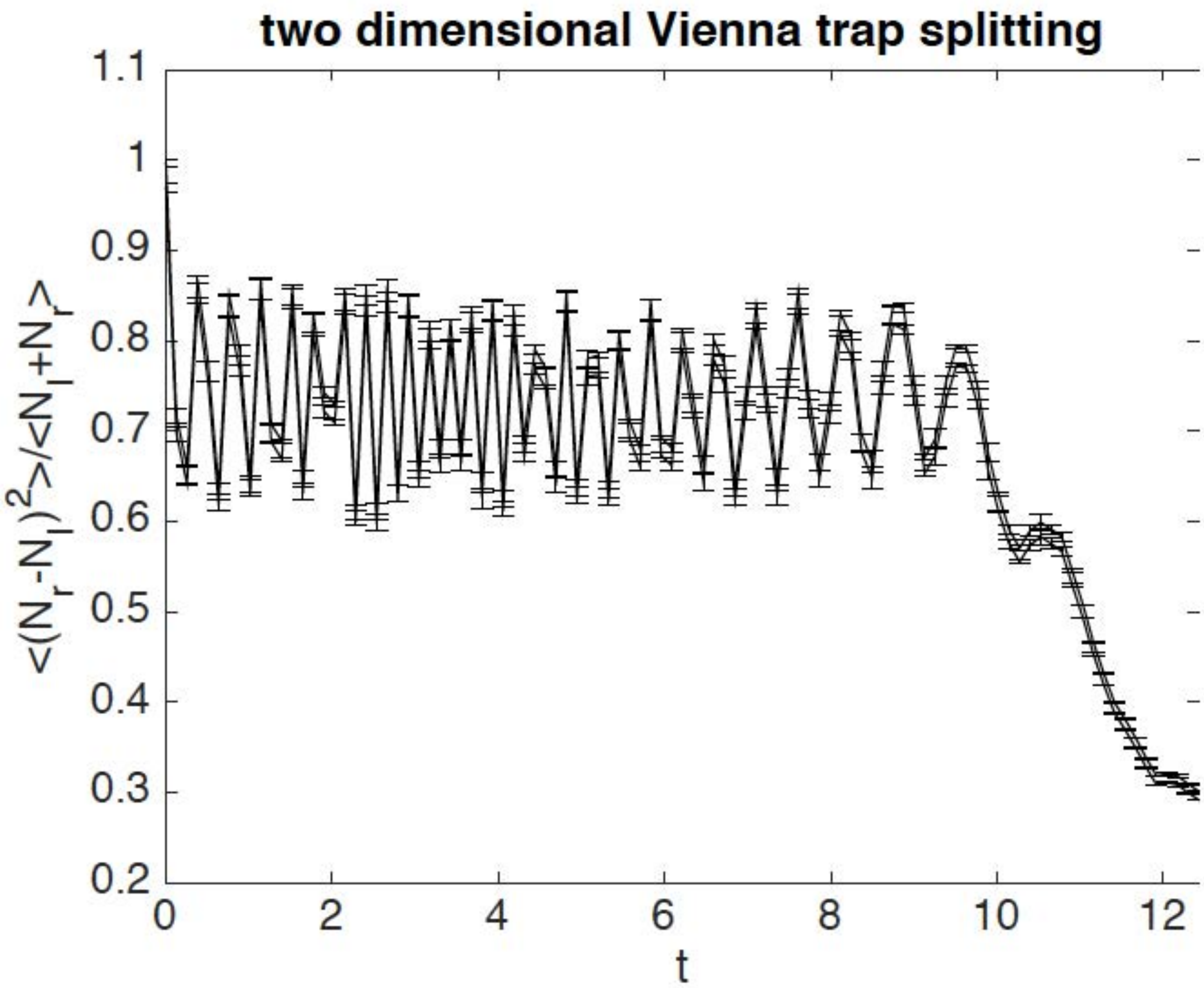
Simulation of condensate splitting

Motivated by Schmiedmayer experiment at Vienna Tech.

- Experiment observes -8dB suppression of correlation noise
- 2D coherent state splitting simulation at SUT
- Simulates a sudden increase in S-wave scattering
- Noise reduction in 2D: -6dB cross-correlation noise

- **Prediction: correlation oscillations**

Simulates 2D splitting, initial coherent state



What about simulating the early universe?

Two methods:

(1) Numerical quantum simulations

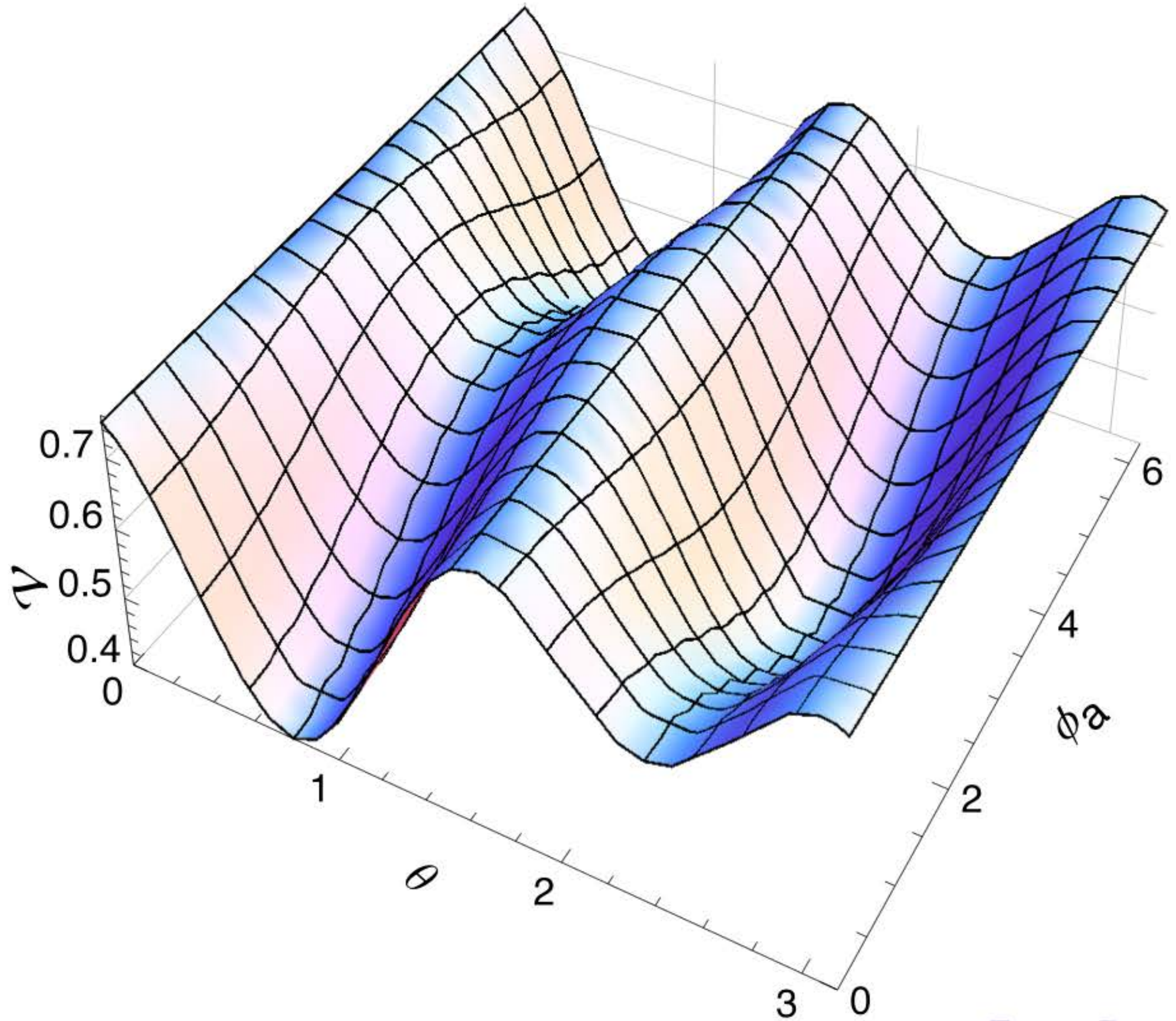
(2) BEC quantum emulation

Early universe quantum simulation

^{41}K Feshbach resonance

- zero inter- state scattering length at 685.7 G
  - nearly equal self-interactions,
  - unknown loss rates (can be estimated)
  - resonance not yet observed

Potential well with microwave coupling



Relativistic field theory at c=1 cm/s!

We introduce the quantum partition function
$\mathscr{Z} = \int \mathscr{D}(\psi^*, \psi) e^{-S[\psi^*,\psi]}$ where:

$$S[\psi^*, \psi] = \int d\mathbf{s} [\psi^*_\sigma \partial_\tau \psi_\sigma + H(\psi^*, \psi)]$$

.

- Imaginary time: $\mathbf{s} = (\tau, \mathbf{r})$, $\tau = it/\hbar \in [0, \beta]$
- $\psi_\sigma(\tau, \mathbf{r})$ is a complex field subject to the periodic boundary condition $\psi_\sigma(\beta, \mathbf{r}) = \psi_\sigma(0, \mathbf{r})$.
- First find static solution to identify vacua. This amounts to replacing $\psi_\sigma = \psi_0 = \mathrm{const}$ in the saddle-point approximation $\delta S / \delta \psi_\sigma = 0$.

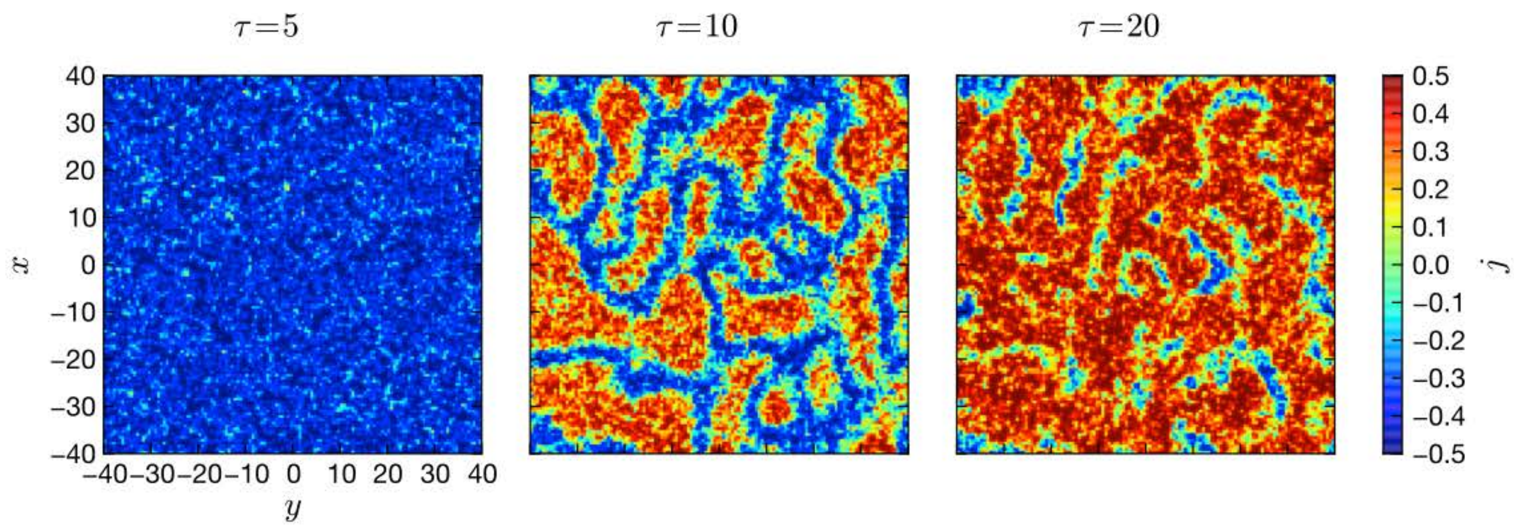
Equivalent Sine-Gordon equation

$$\psi_1 = u e^{i(\phi_s+\phi_a)/2}\cos(\theta)$$
$$\psi_2 = u e^{i(\phi_s-\phi_a)/2}\sin(\theta),$$

- Canonical momentum: $\pi = \partial_\tau \phi_a / 4\gamma_{sa}$,
- Commutators: $\left[\phi_a(\zeta), \pi(\zeta')\right] = i\delta^D(\zeta - \zeta')$.
- Sine-Gordon equation:

$$\nabla^2 \phi_a - \partial_{\zeta_0\zeta_0}\phi_a + \tilde{\alpha}\sin\phi_a = 0$$

3D Universe: BEC simulations



Plot of 3D density evolution. Coupling $\tilde{\nu} = 0.1$.
Opanchuk et. al, Annalen der Physik **525**, 866 (2013).

The original: Coleman's 'false vacuum'

PHYSICAL REVIEW D VOLUME 15, NUMBER 10 15 MAY 1977

Fate of the false vacuum: Semiclassical theory*

Sidney Coleman
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 24 January 1977)

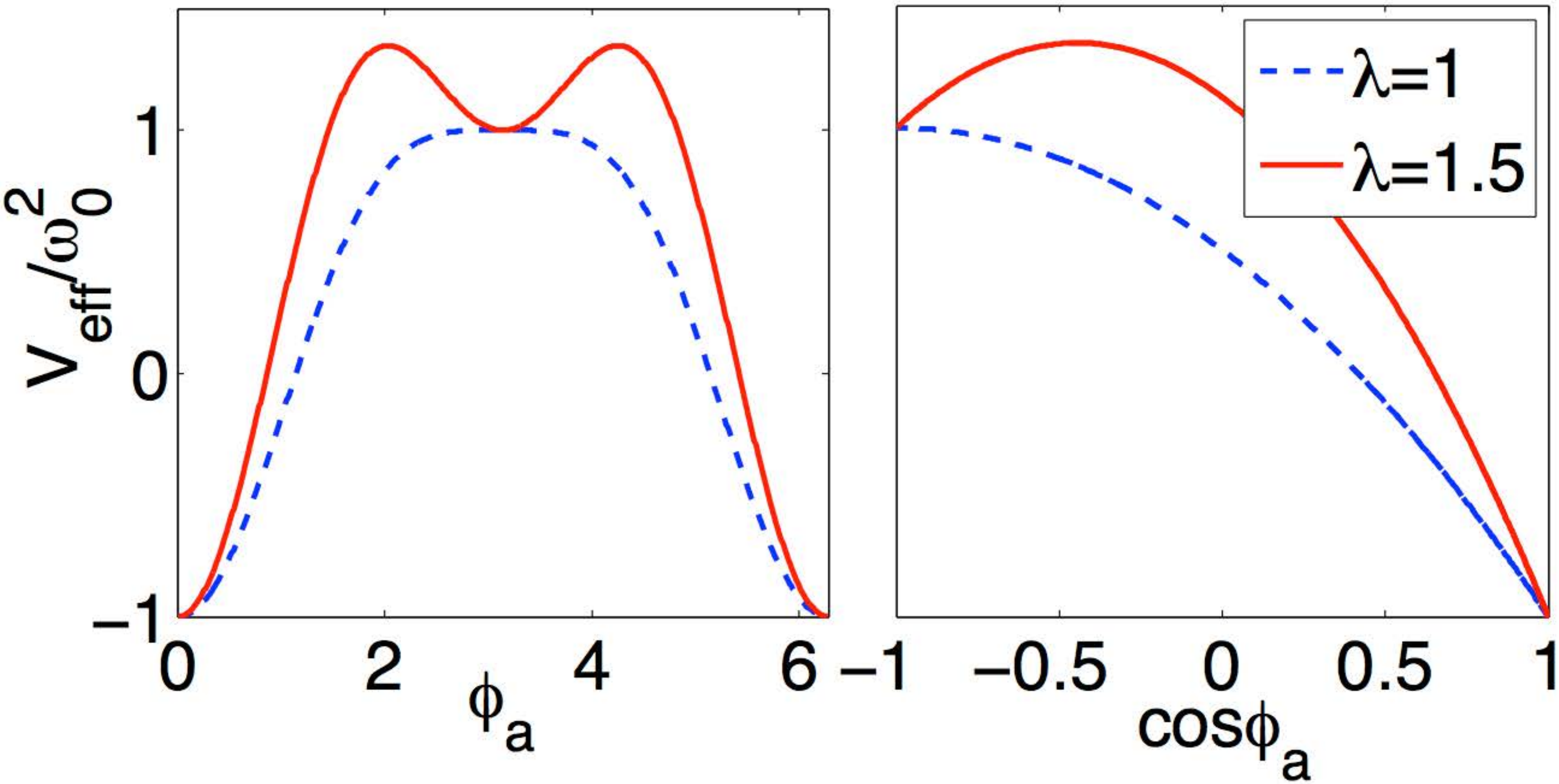
It is possible for a classical field theory to have two homogeneous stable equilibrium states with different energy densities. In the quantum version of the theory, the state of higher energy density becomes unstable through barrier penetration; it is a false vacuum. This is the first of two papers developing the qualitative and quantitative semiclassical theory of the decay of such a false vacuum for theories of a single scalar field with nonderivative interactions. In the limit of vanishing energy density between the two ground states, it is possible to obtain explicit expressions for the relevant quantities to leading order in h; in the more general case, the problem can be reduced to solving a single nonlinear ordinary differential equation.

What about metastability: local minimum?

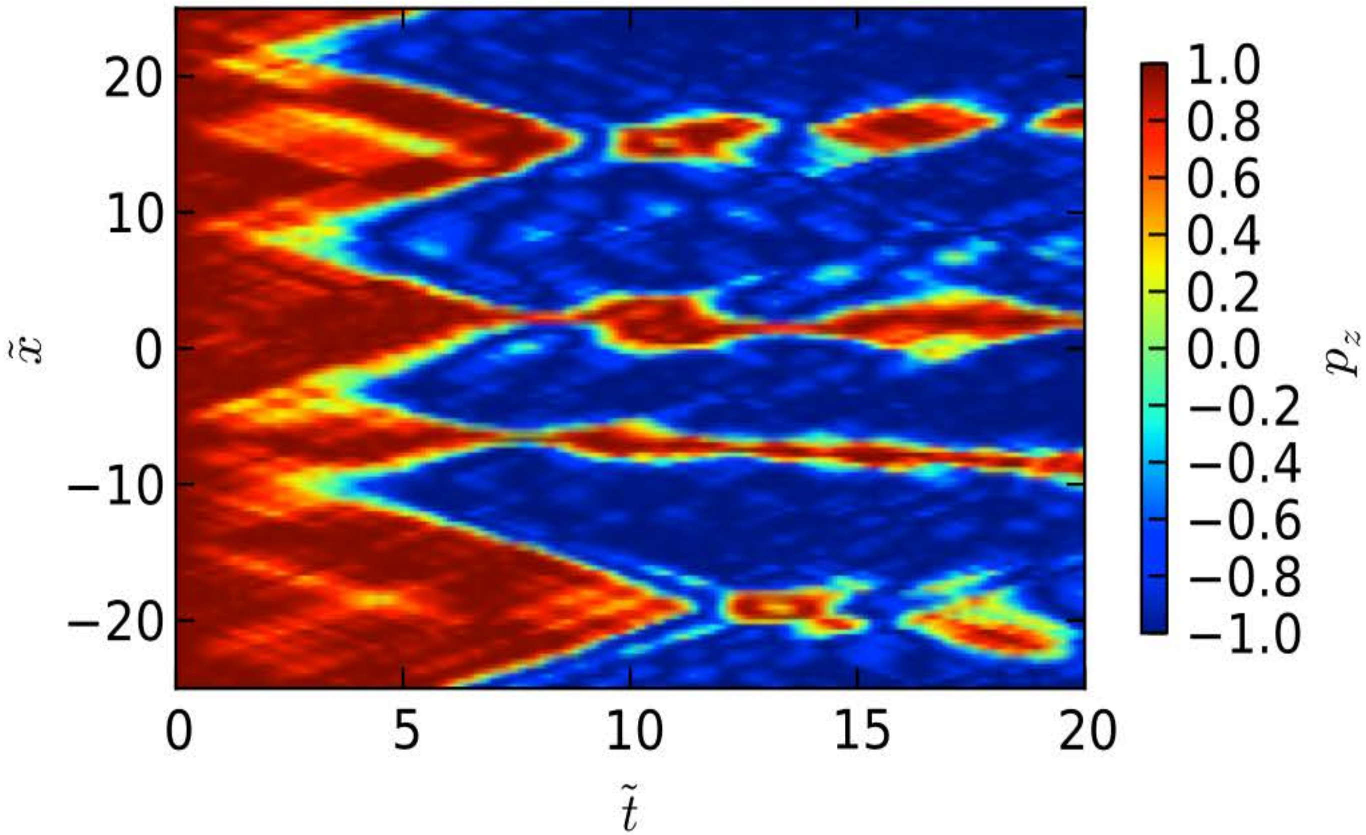
- By varying the tunnel coupling ν periodically in time, it is possible to establish the unstable vacuum at $\phi_a = \pi$
- Gives the Coleman model
- Microwave coupling $\nu_t = \nu + \delta\hbar\omega\cos(\omega t)$, where frequency of oscillations $\omega \gg \omega_0 \equiv 2\sqrt{\nu g \rho_0}/\hbar$
- $V(\phi) \to V_{\text{eff}}(\phi_0) = -\omega_0^2\left[\cos\phi_0 - 0.5\lambda^2\sin^2\phi_0\right].$
- $\lambda = \delta\hbar\omega_0/\sqrt{2}\nu > 1$

O. Fialko et. al., Europhysics Letters **110**, 56001 (2015).

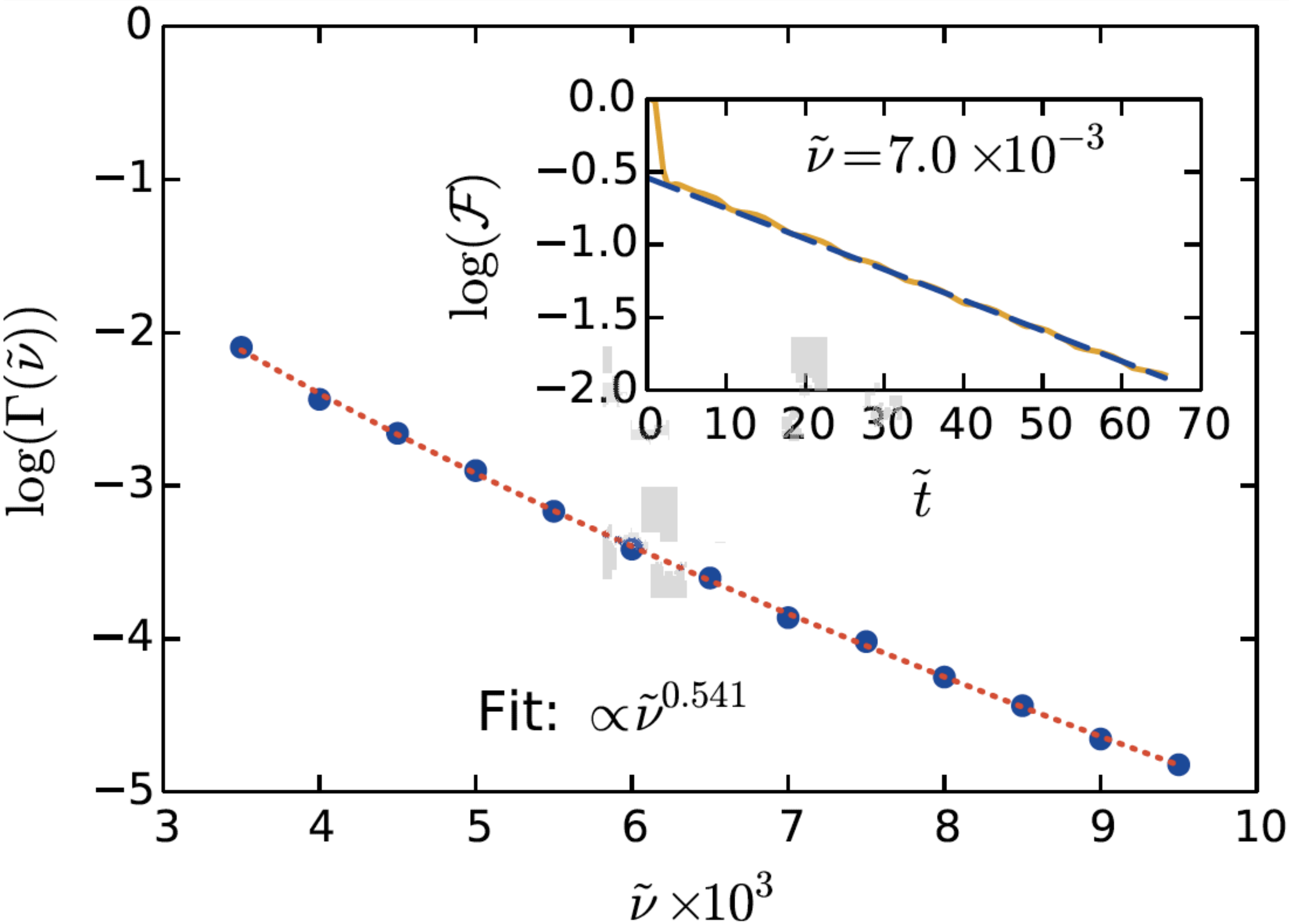
Effective potential



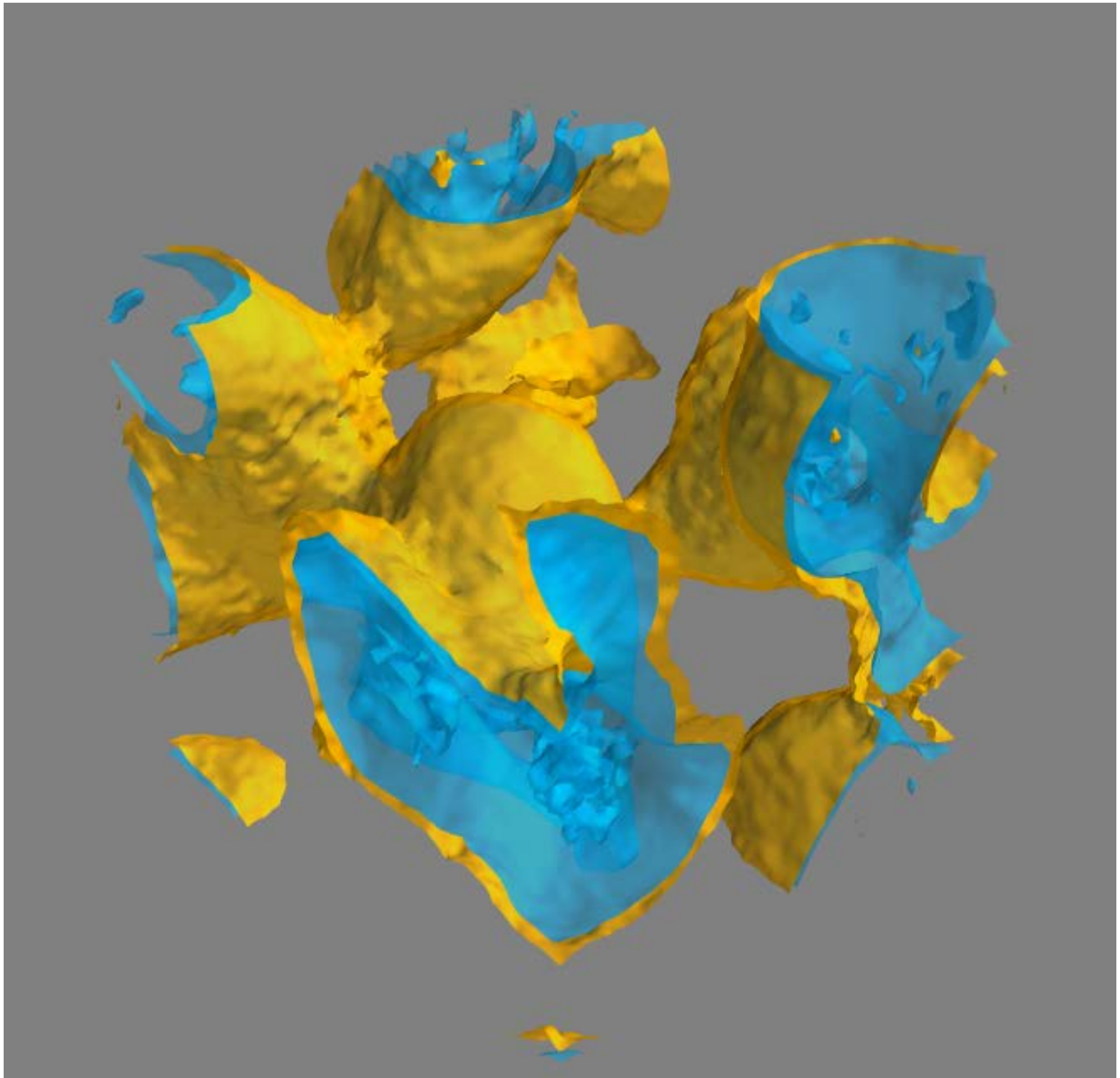
Vacuum bubbles expand at light-speed



Tunneling rate depends on well-depth



3D bubbles



Metastability in 3D : bubble universes

Early universe models

- What is now being measured?
- Cosmologists are measuring third order nonlinear correlations
- Observed correlations mostly gravitational
- Is there a quantum noise contribution?

Work in progress

- Effect of finite trap?
- Effects of finite temperatures?
- Removing TW approximation

Conclusions

- **Blooming cosmology: CMB, BICEP2.**
- **Inflation might be correct: can we test it?**
- **Time to bring cosmology into the laboratory**
- **Analog quantum/numerical simulator with BEC!**
- O. Fialko, et. al. Europhys. Letts **110**, 56001 (2015).
- **What can we measure experimentally?**