数理モデリングで未来を創る
～次世代の電気材料へ～

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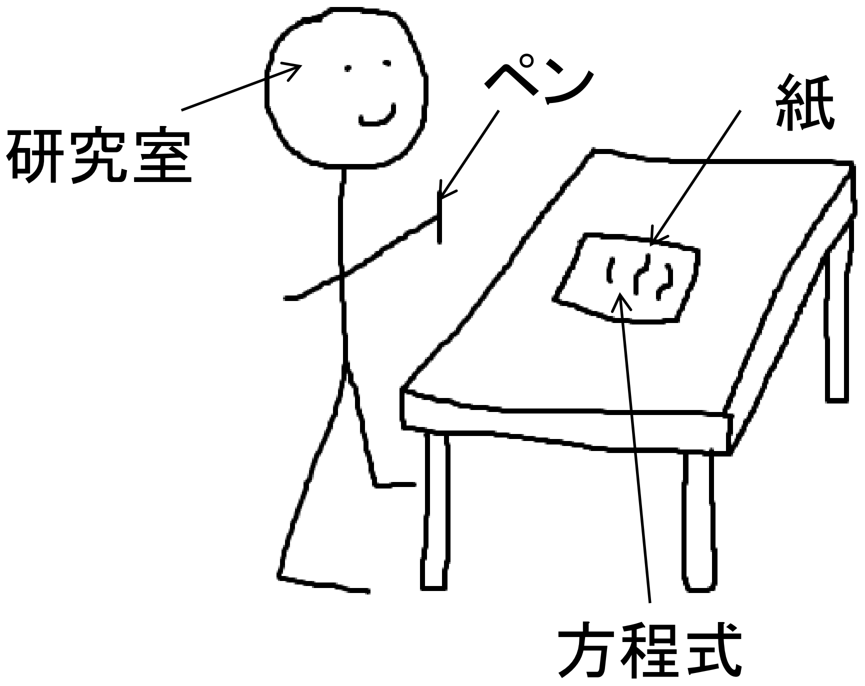
PRESTO

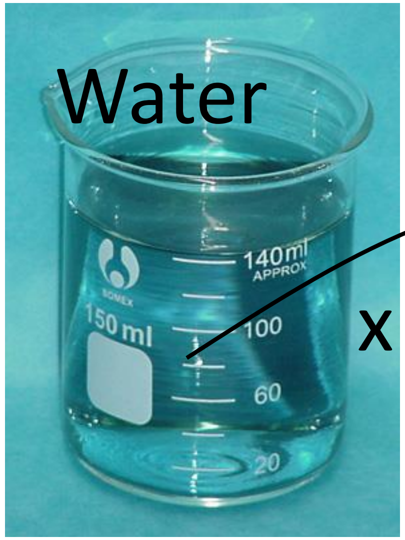


I am a **chemist** and an **applied mathematician.**

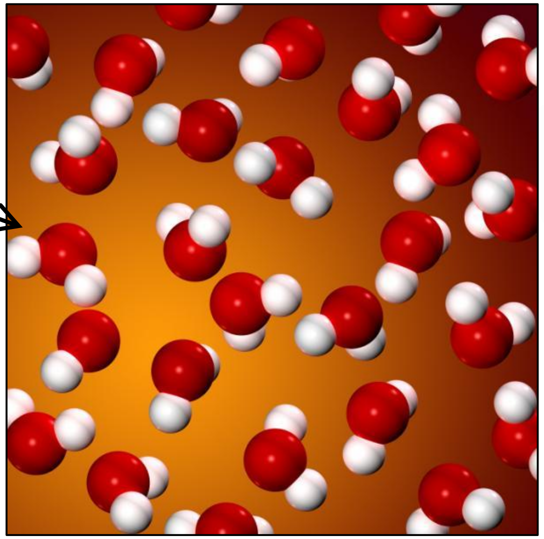
Chemistry = Science of creating new materials using molecules

Applied Mathematics = Science of abstracting natural phenomena

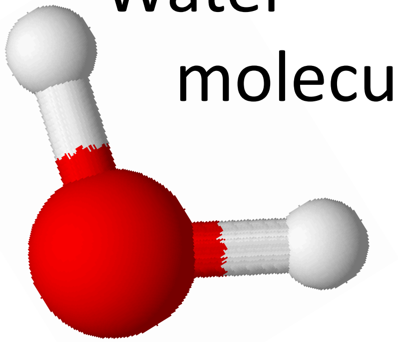
Molecules make up matter



x 10000000000



Water molecule



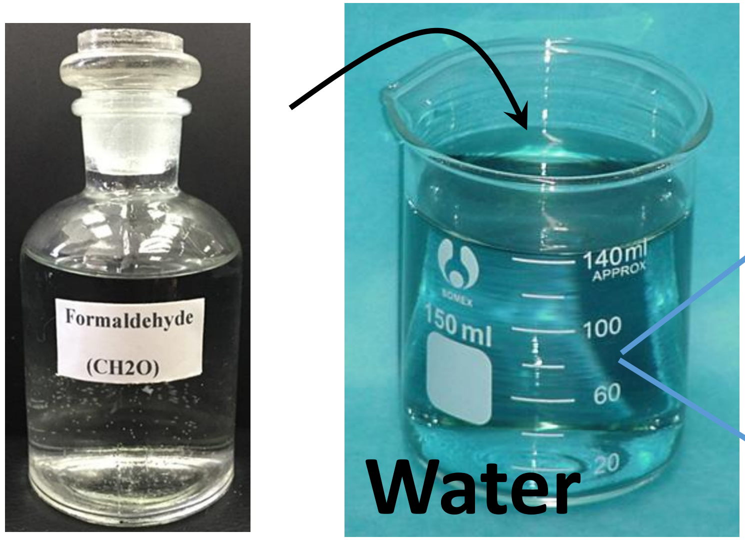
9.6×10^{-9} cm
(0.0000000096 cm)

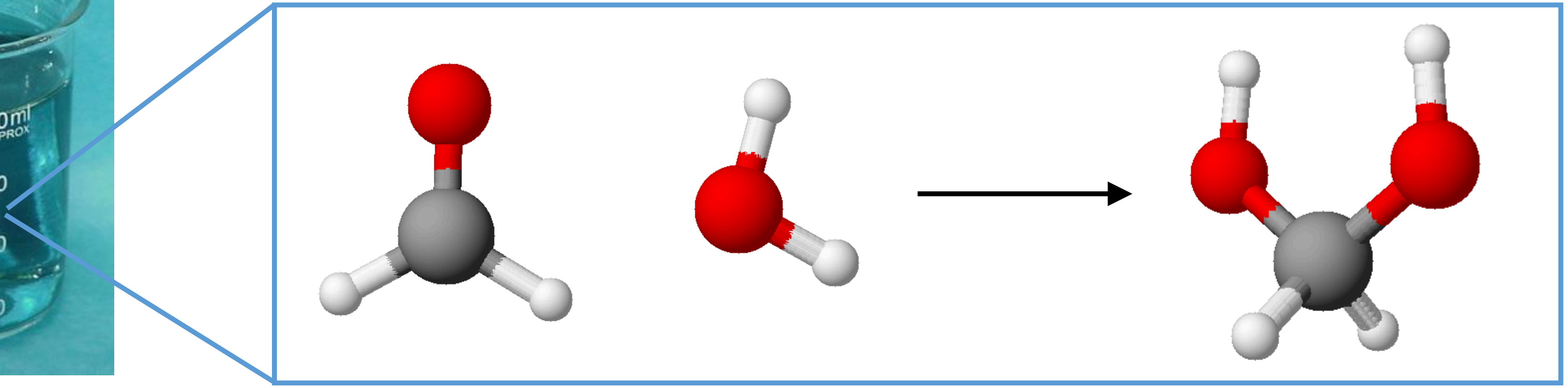
How to make new materials using molecules?

1. Chemists mix molecules together.

2. Molecules **interact** with each other

3. **Sometimes, interaction produces new material**





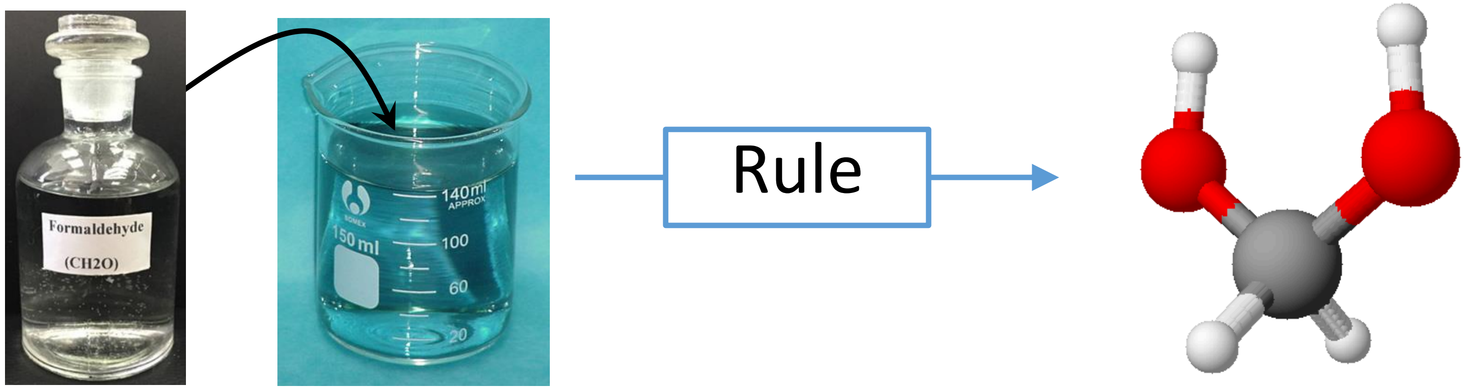
Chemists **can control this step**

Chemists **cannot control** this step!

Key point: Before mixing the molecules together, chemists must **know the rules** for how the molecules will interact.

What is a rule?

A ‘rule’ predicts what happens ***without* doing an experiment.**



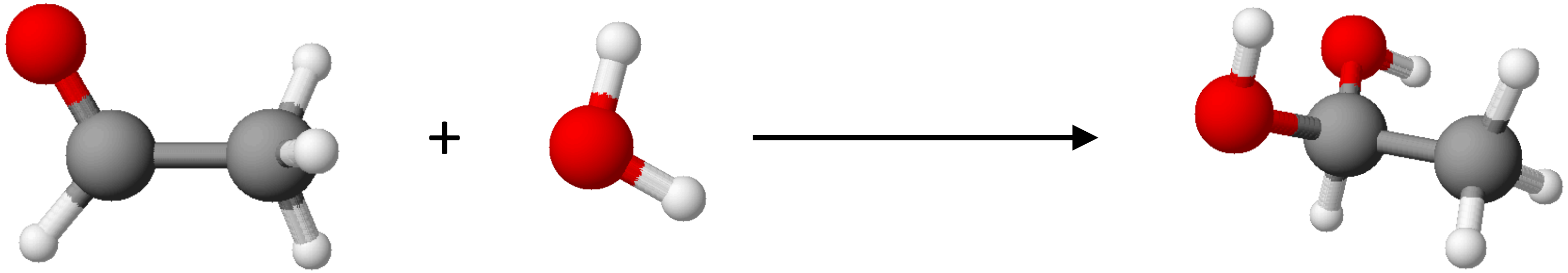
How to deduce new rules?

This is very difficult.

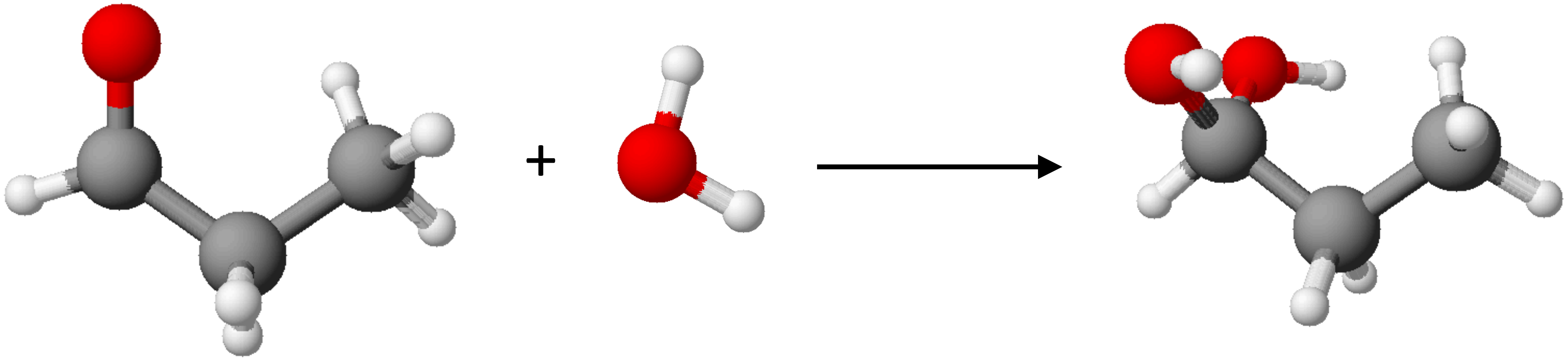
A chemist needs to do **many, many experiments**…

Example: Let’s deduce a rule!

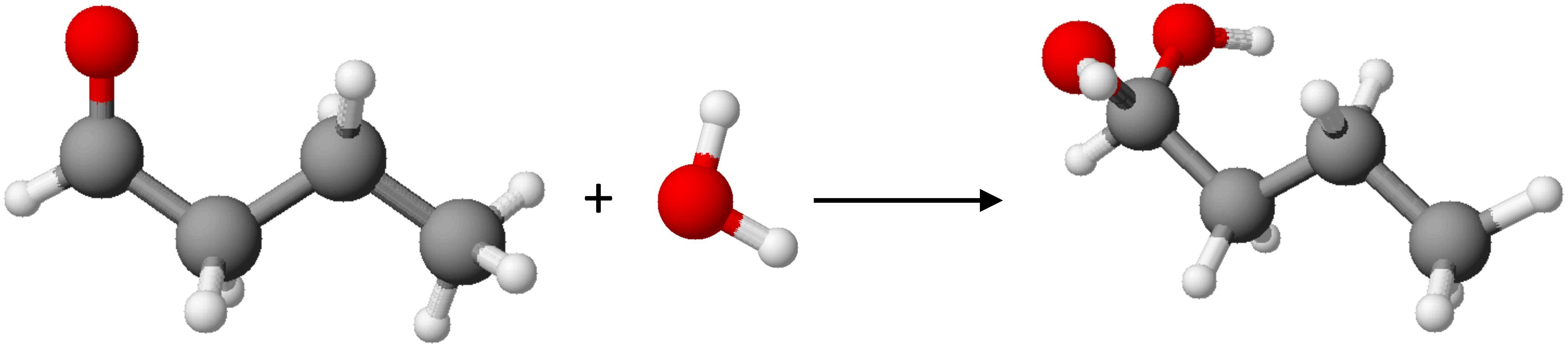
Experiment 1



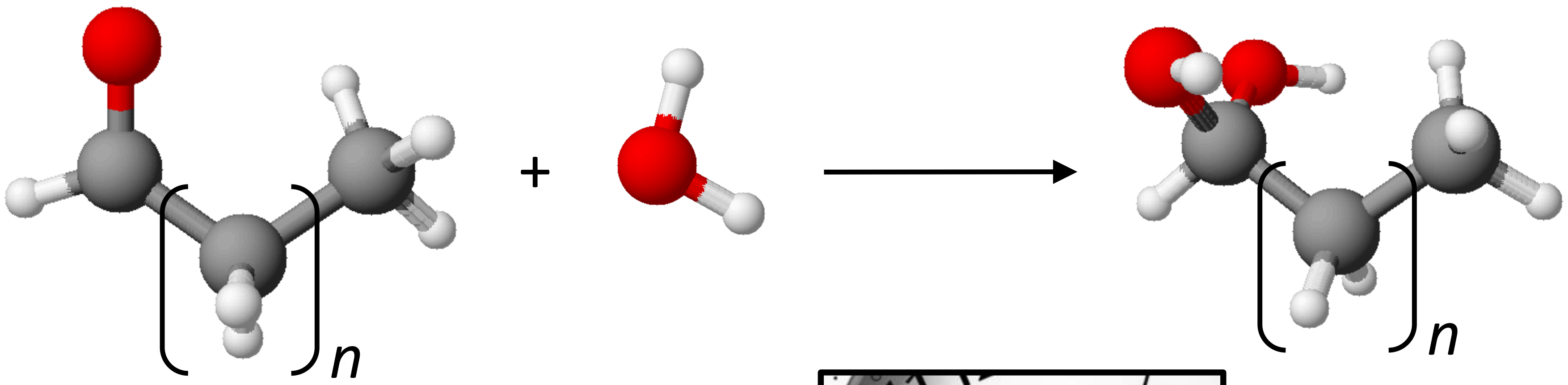
Experiment 2



Experiment 3



Rule (n $\geq$ 1)



Why is it hard to deduce rules?

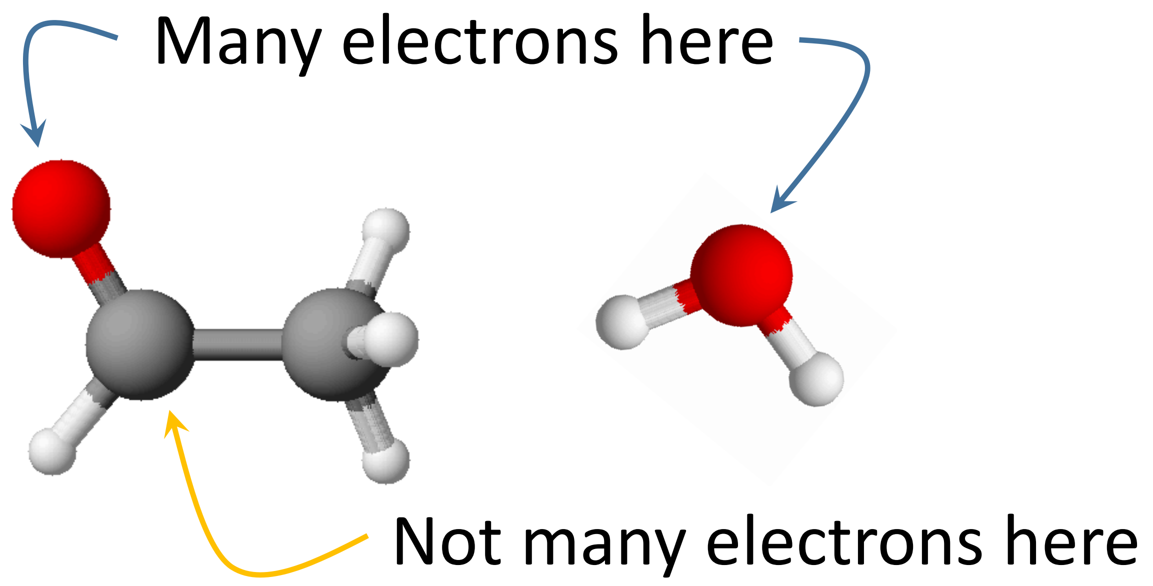
Chemist must do **many, many experiments.**

- Experiments take a **long time**.
- Experiments are often **dangerous.**
- Experiments cost **a lot of money**.

But there is **another way** to deduce rules… **mathematical modelling!**

Mathematical Modeling Process

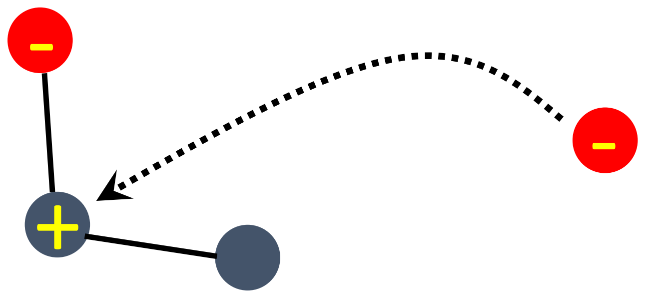
1. Identify the **key parts** of the real situation:



2. **Abstract** the real situation

3. Apply the **laws of physics**

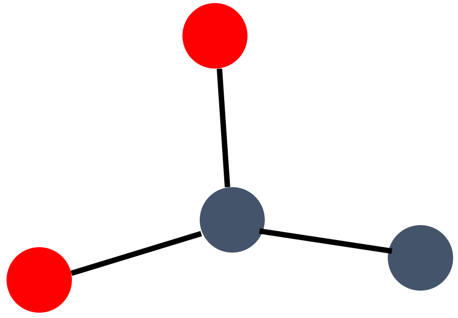
→ Tells us the rule



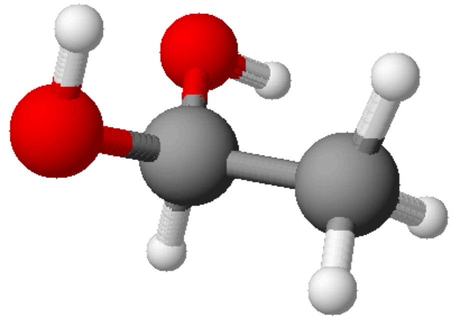
Physics: Positive attracts negative.

4. Compare to **experiment**

Similar → Rule seems reasonable.



Experiment

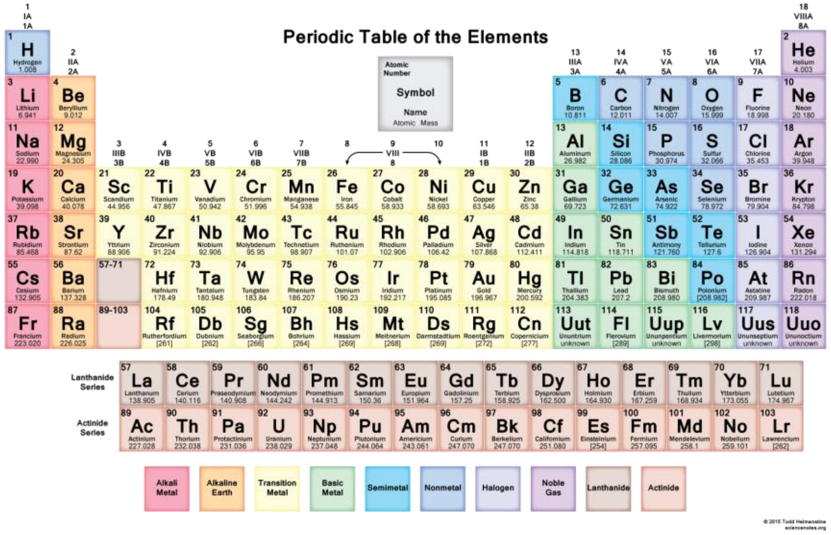


Key Steps for Mathematical Modeling

1. Identify the key parts of the real situation

2. Abstract the real situation

Need a good knowledge of the situation!



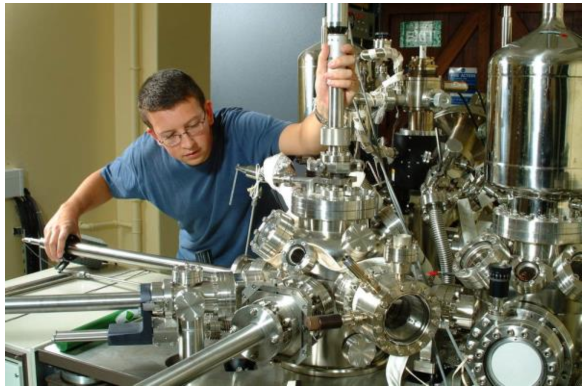
3. Apply the laws of physics

Need to be good at mathematics!

$$\frac{dX_t}{dt} = f(X_t) + b\frac{dW_t}{dt}$$

4. Compare to experiment

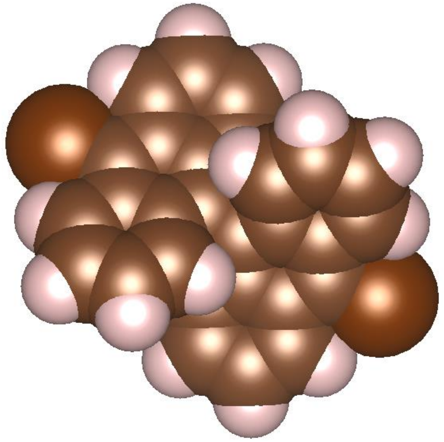
Need friends who can do experiments!



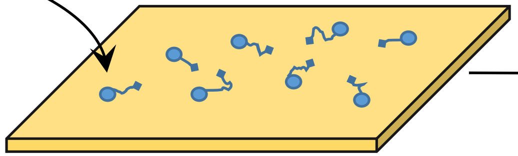
Mathematical modelling for next-generation electrical materials

Experiment by P. Han, T. Hitosugi (2014, Tohoku University)

Di-bromo bi-anthryl (DBBA) molecule

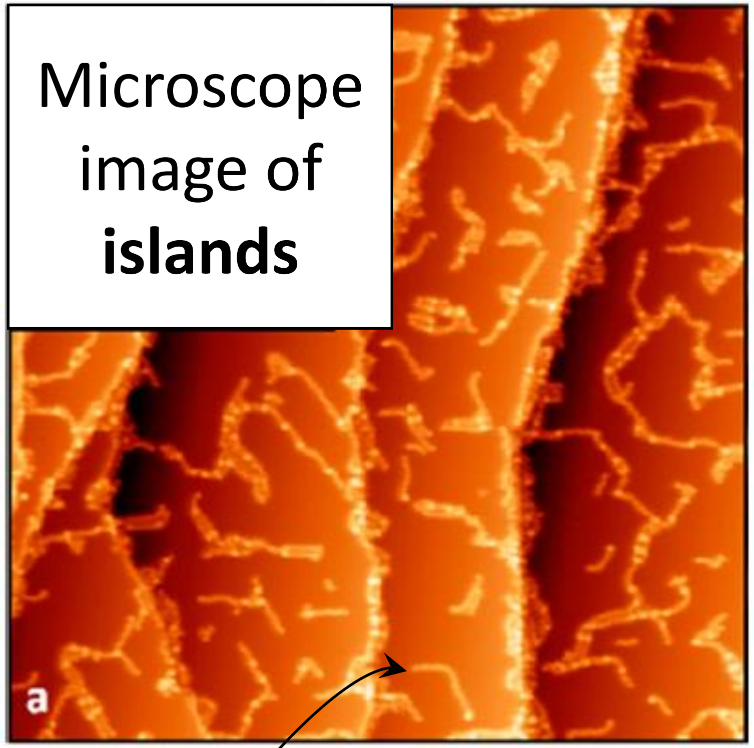


Place on copper surface



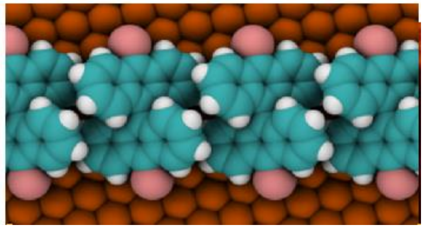
Molecules diffuse on surface

1.380×10^{-7}m



1.380×10^{-7}m

‘Molecule island’

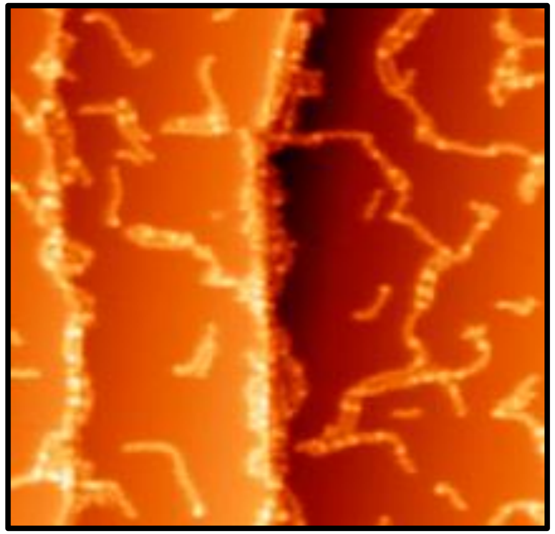


What happened?

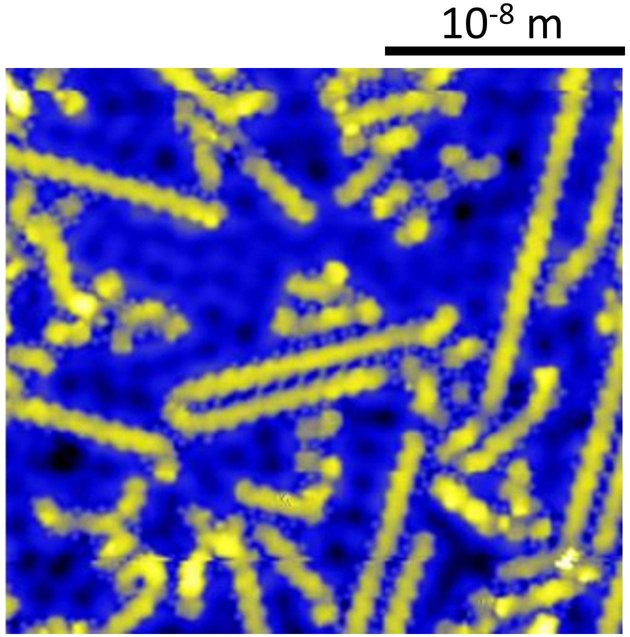
The molecules **assembled into islands**.

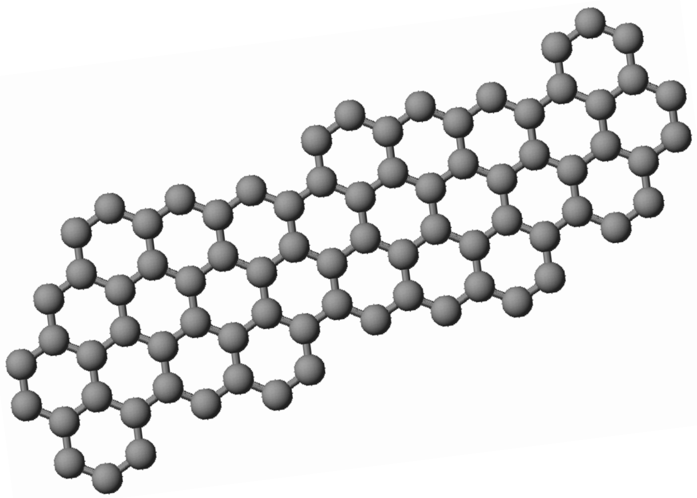
This phenomenon is called **molecular self-assembly.**

Molecular self-assembly is **very useful**…



Heat
(800 ^{o}C)





Islands turn into **graphene**

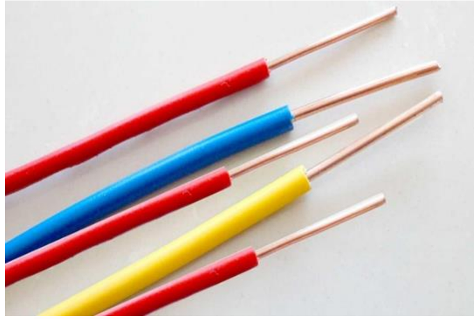
What is special about graphene?

Graphene has **extremely high conductivity** (more than 2000 x silicon)

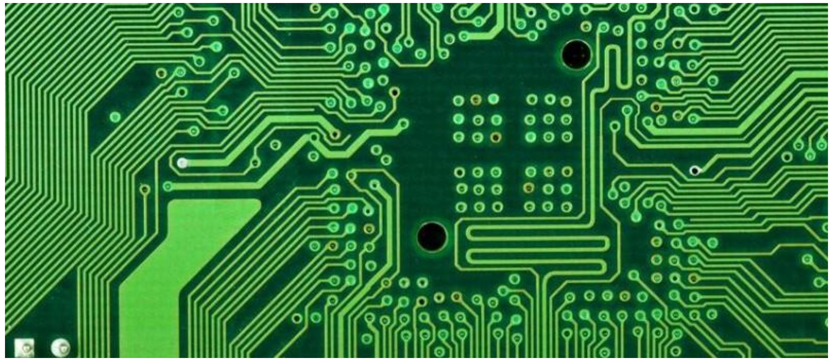
Dream: Create **real electronics** using graphene (e.g., extremely fast computers).

How might we make real electronics using graphene?

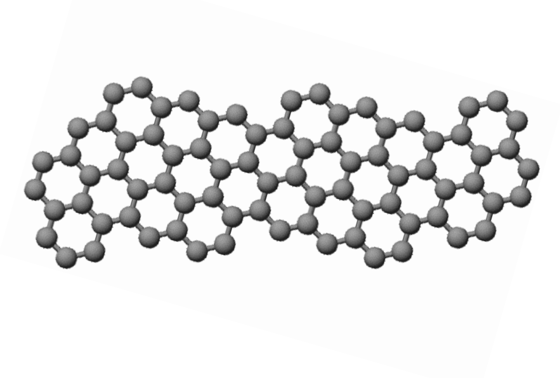
Wire:



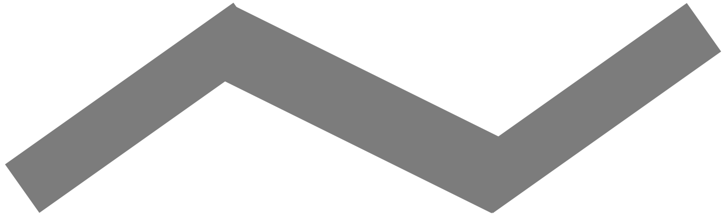
Electronic circuit:

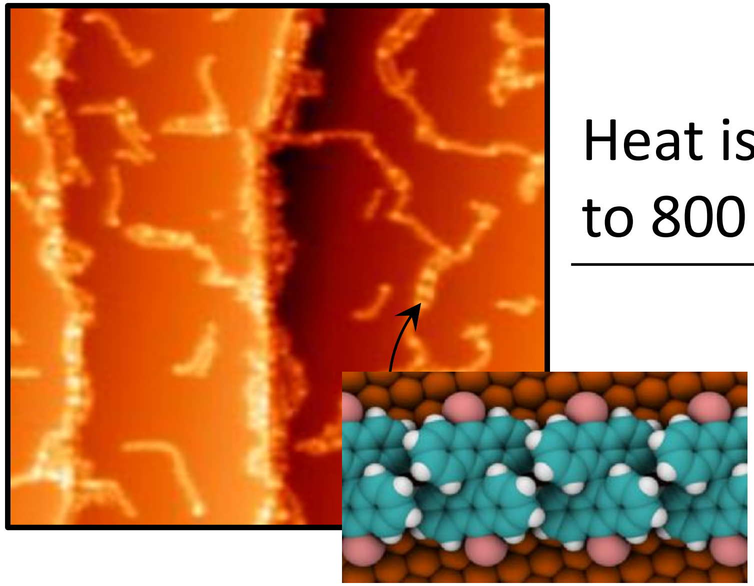


Wire-shaped graphene



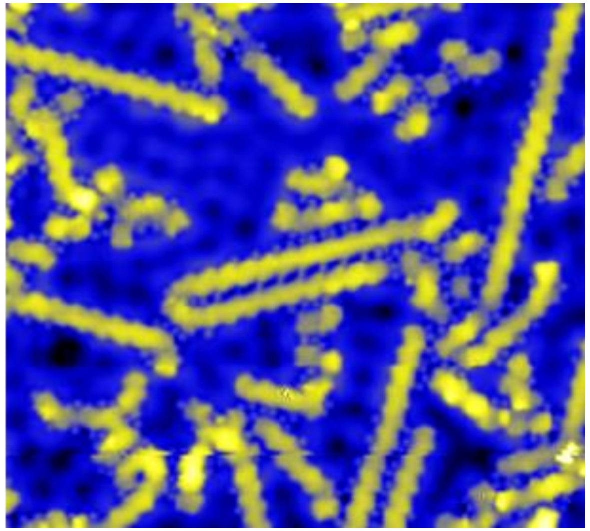
Non-straight wires





Wire-shaped island

Heat islands to 800 ^{o}C →



Wire-shaped graphene

How to make?

Control island structure

→ Control graphene

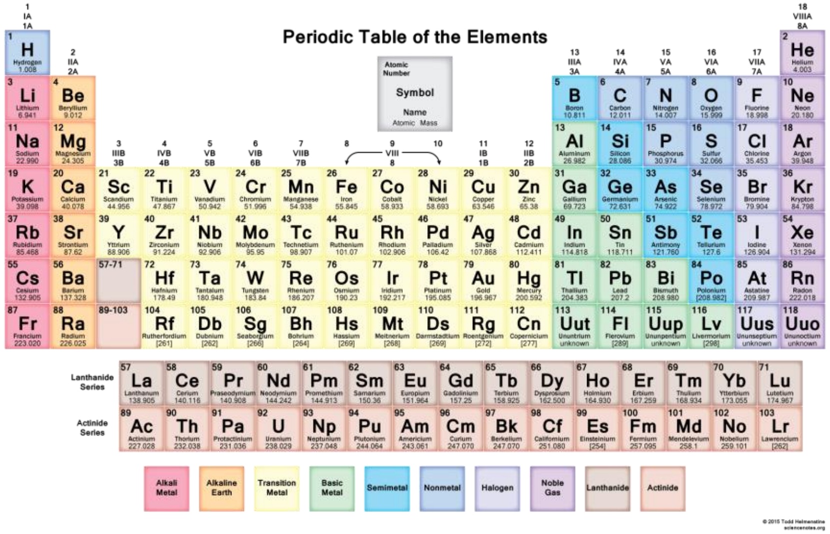
But, **nobody knows the rules** for island formation.

Let’s try mathematical modeling!

Mathematical modeling for island formation

1. Identify the key parts of the real situation.

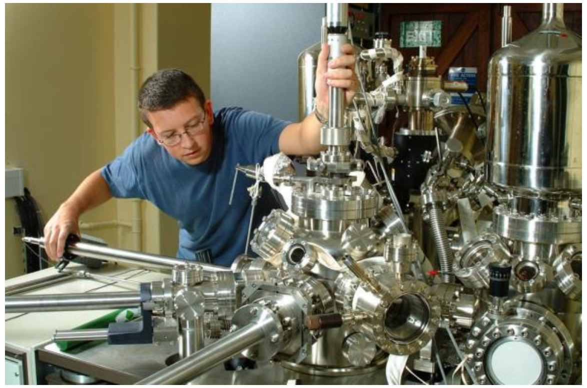
2. Abstract the real situation



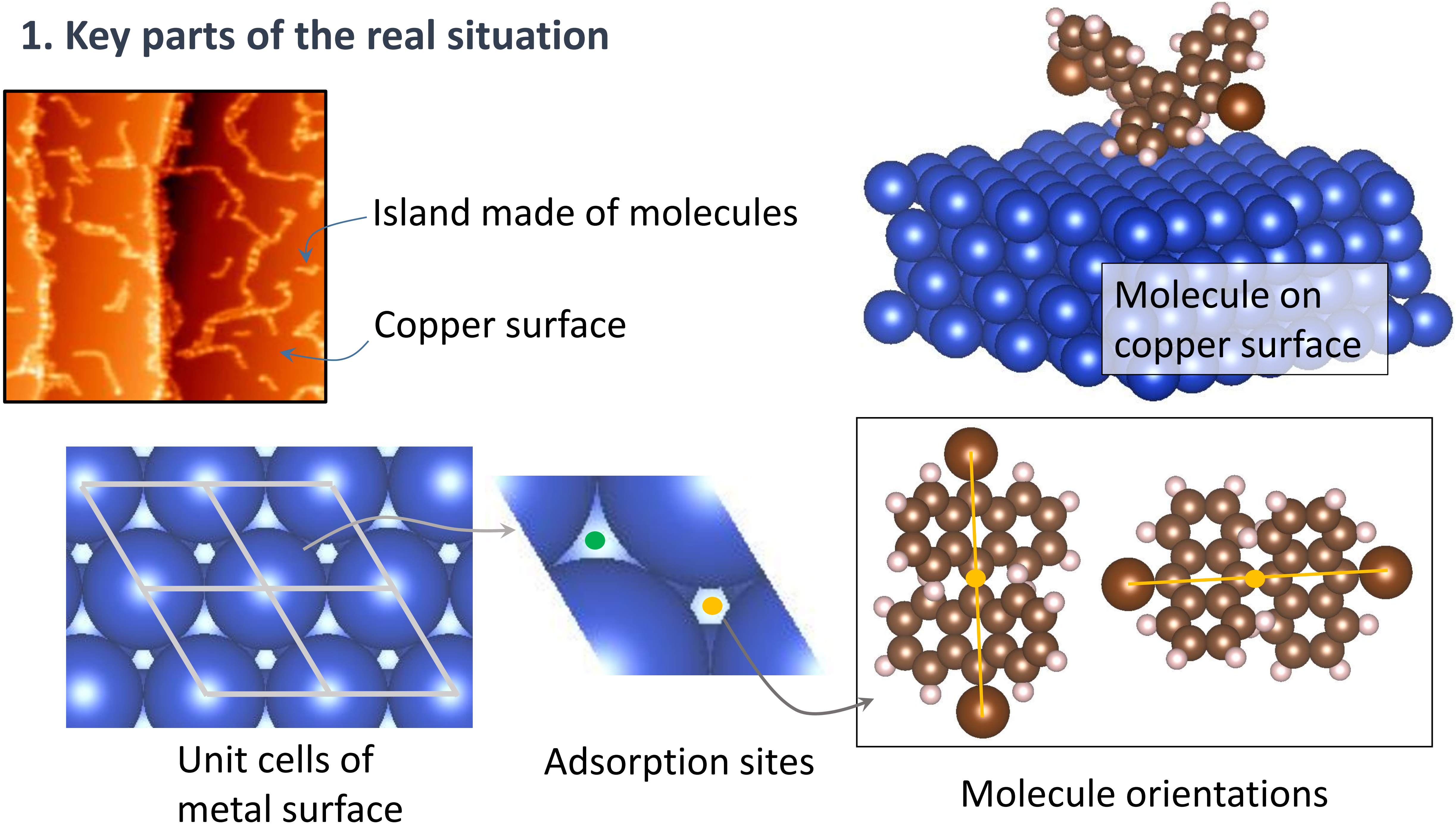
3. Apply the laws of physics

$$\frac{dX_t}{dt} = f\left(X_t\right) + b\frac{dW_t}{dt}$$

4. Compare to experiment

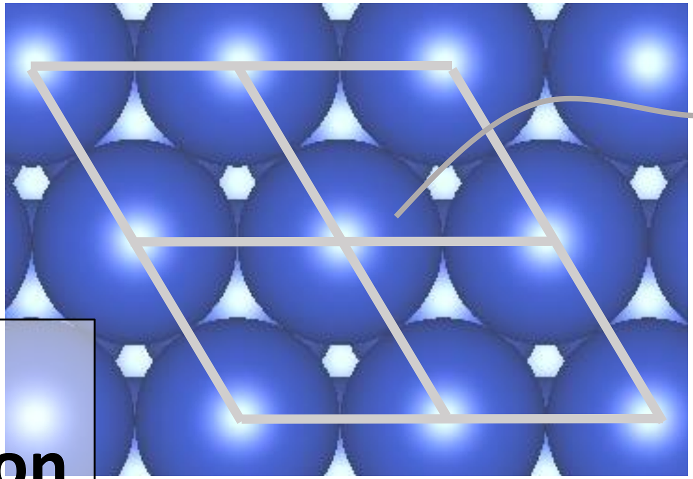


1. Key parts of the real situation

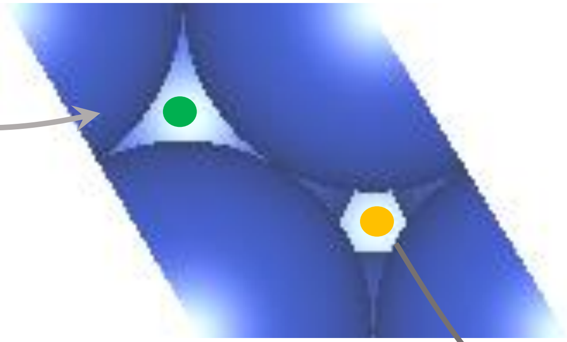


2. Abstract the real situation

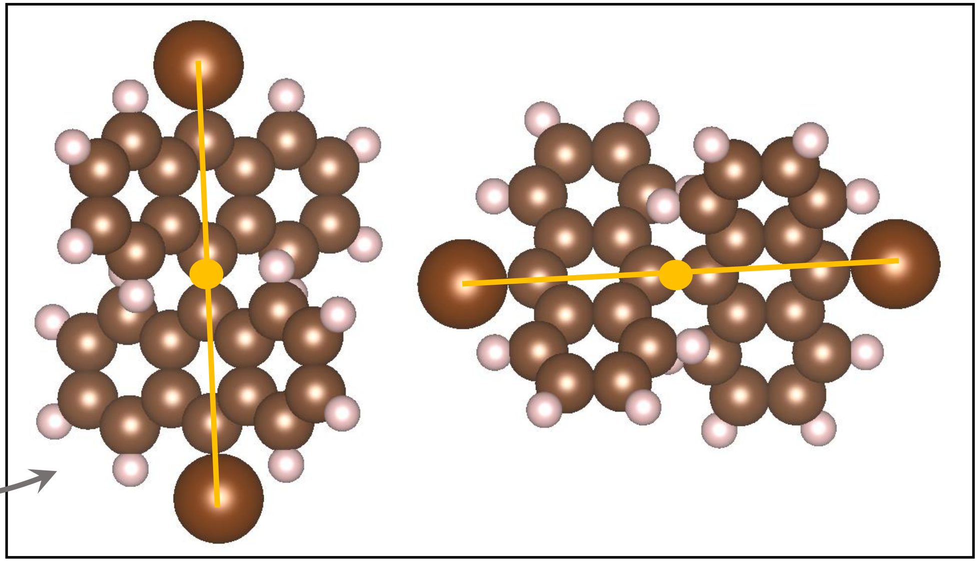
Real situation



Unit cells of metal surface

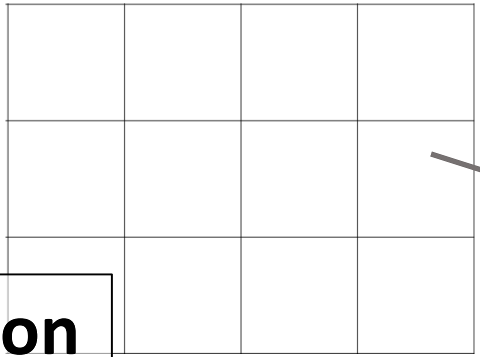


Adsorption sites



Molecule orientations

Abstraction



'Cells' $C_1, C_2, \ldots, C_M$

$\{ \sigma_1 \quad \sigma_2 \}$

'Colours'

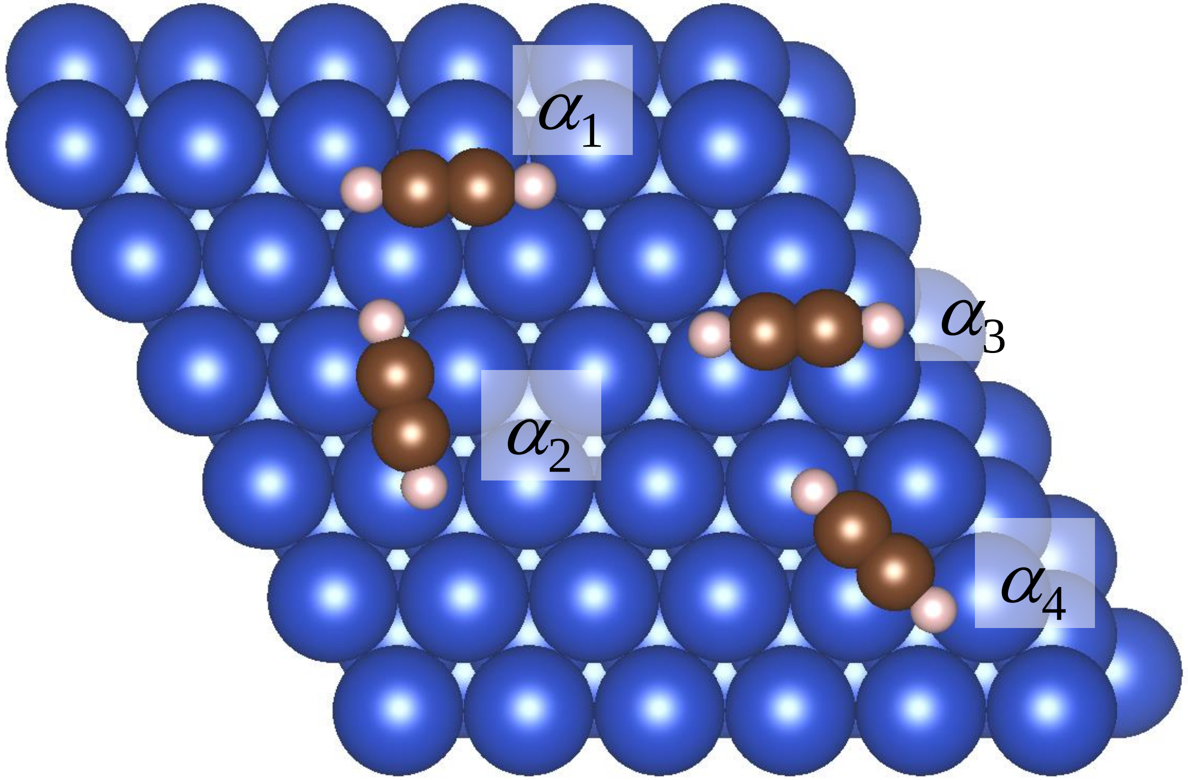
$\sigma_1, \sigma_2, \ldots, \sigma_k$

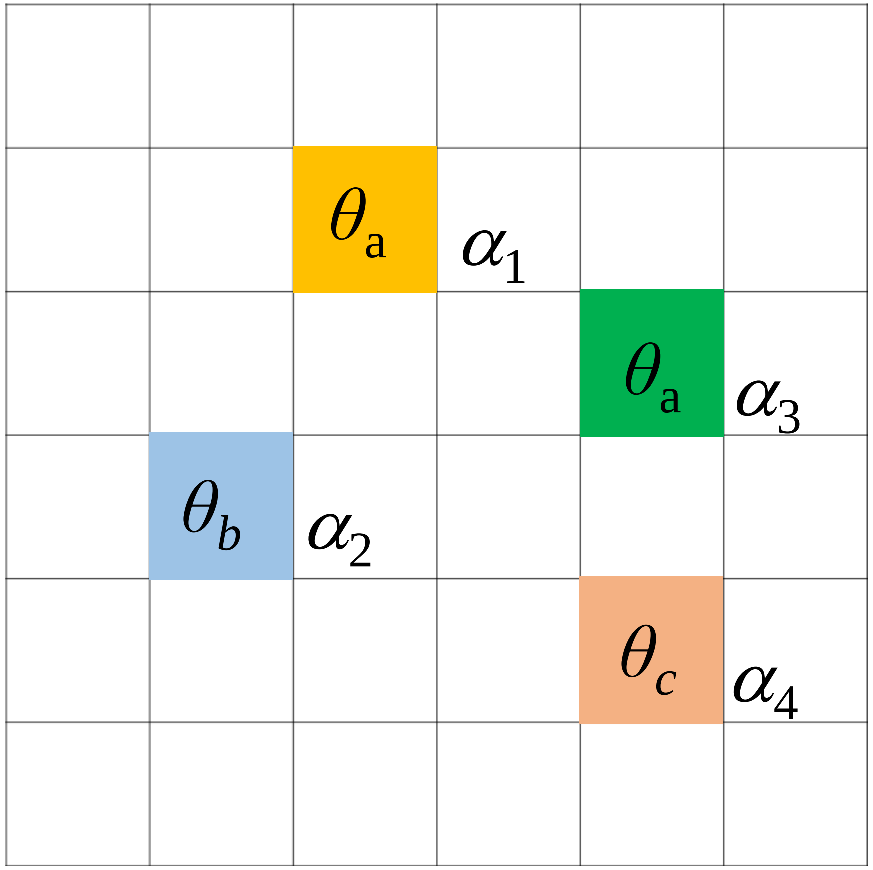
$\{\theta_{2,1}, \theta_{2,2}\}$

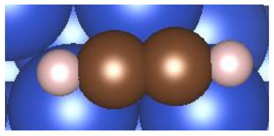
'Shades'

$\theta_{i,1}, \theta_{i,2}, \ldots, \theta_{i,n(i)}$

Reality – abstraction correspondence

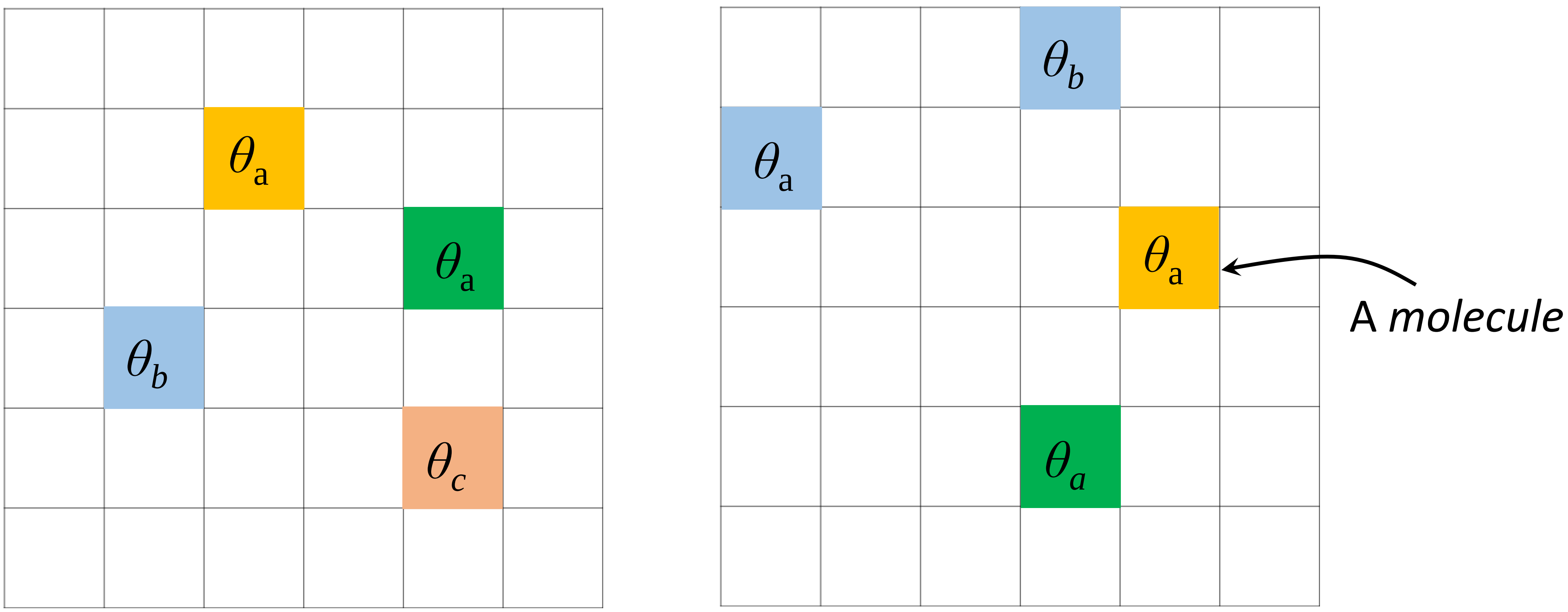






= HCCH molecule

A *configuration* is any choice of *N* cells, colours, and directions



Two configurations (*N* = 4, four colors, three shades θ_a, θ_b, θ_c)

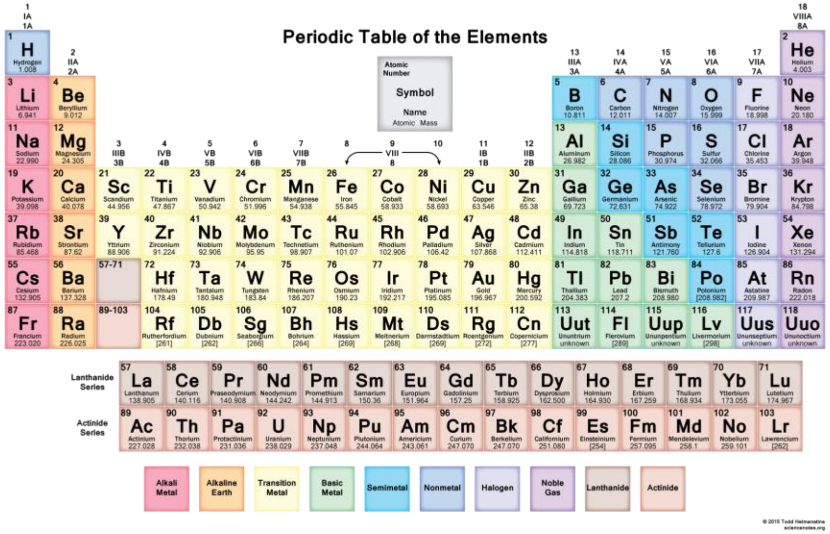
Mathematical modeling for island formation

✓ 1. Identify the key parts of the real situation.

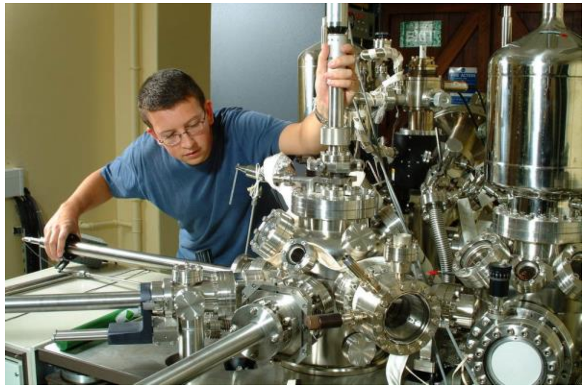
✓ 2. Abstract the real situation

3. Apply the laws of physics

4. Compare to experiment



$$\frac{dX_t}{dt} = f(X_t) + b\frac{dW_t}{dt}$$



Let *c* be a configuration.

Boltzmann distribution law from physics

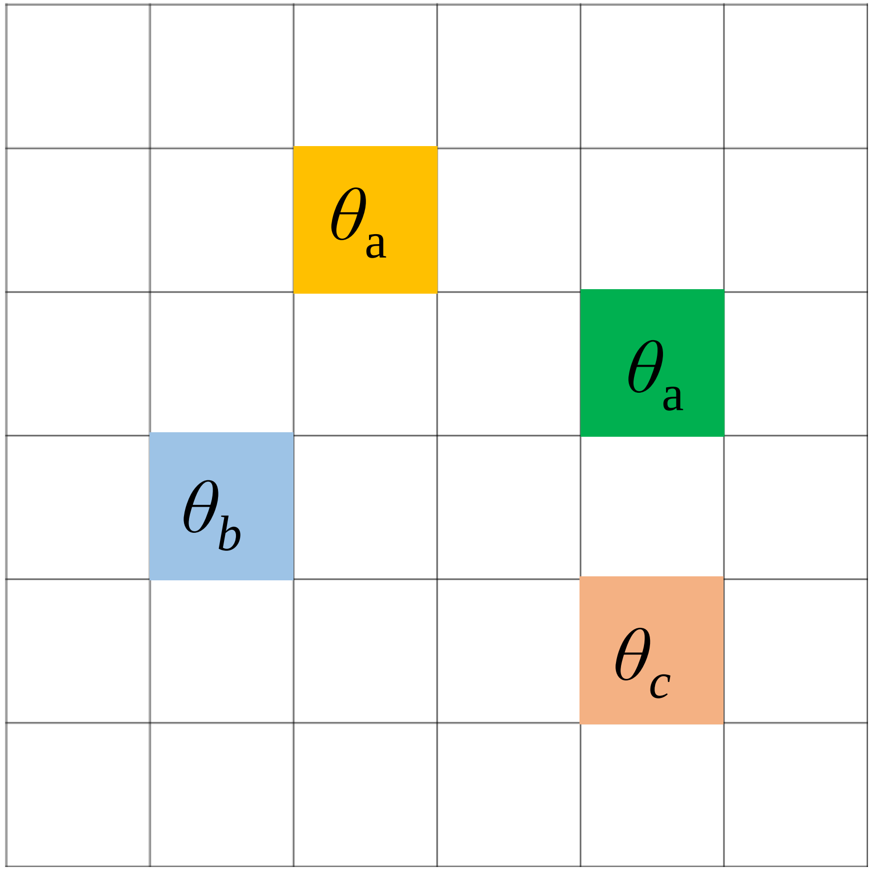
→ probability of configuration *c*

$$p(c) \propto \exp\left(-\varepsilon(c)/k_B T\right)$$

$\varepsilon(c)$ = energy of configuration *c* (we can calculate)

k_B = Boltzmann constant (1.38 x 10^{-23} J K^{-1})

T = Temperature

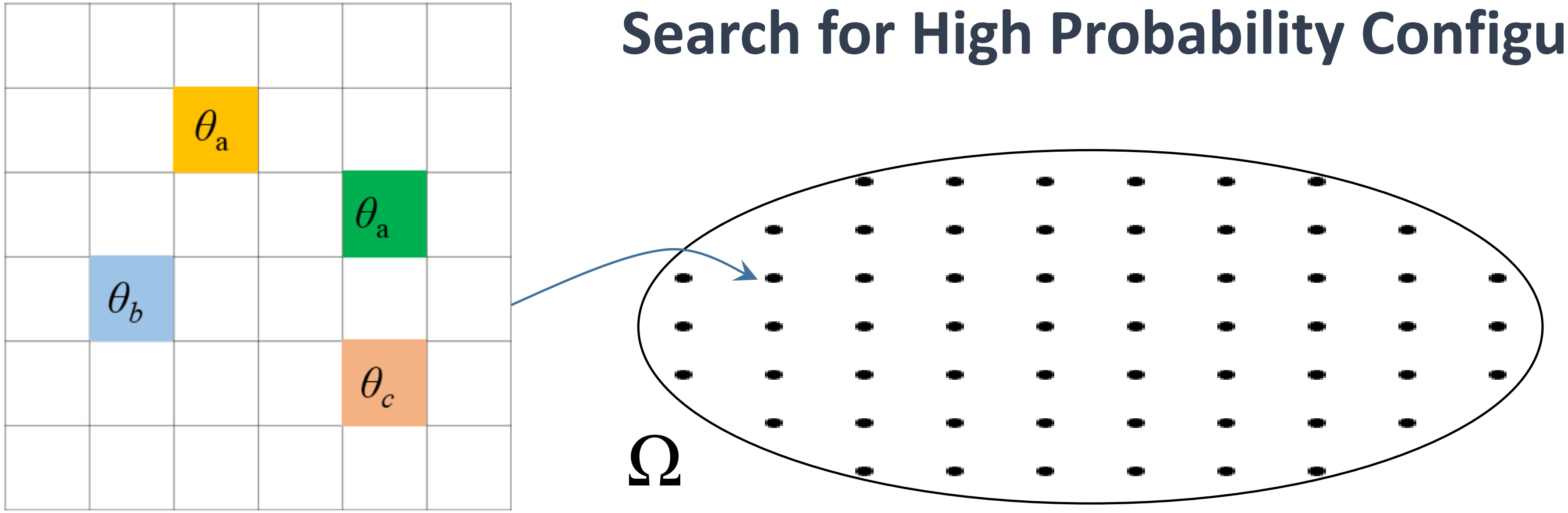


one configuration (*c*)

Mathematical challenge

Find configurations that have high probability.

Search for High Probability Configurations



Ω = configuration space

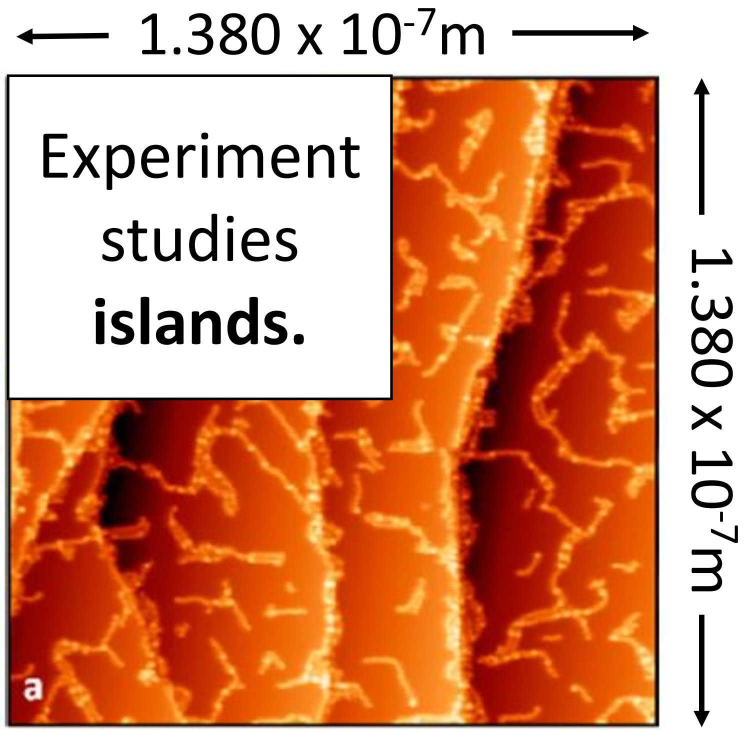
Configuration space is extremely big!

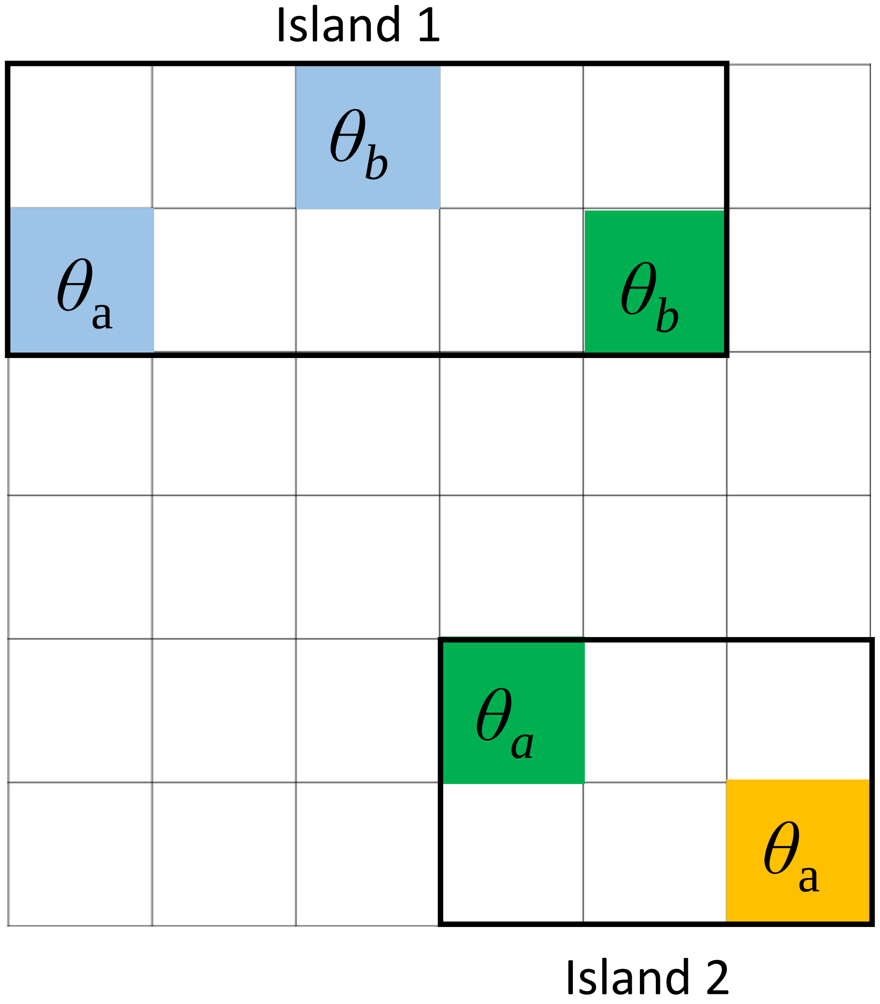
Around 10^{23} configurations (case of 1000 cells with 10 molecules)

Suppose it takes 10^{-6} s for computer to calculate probability of one configuration.

Then $10^{23} \times 10^{-6}$ s $\approx 10^{10}$ years to check probability for every configuration!

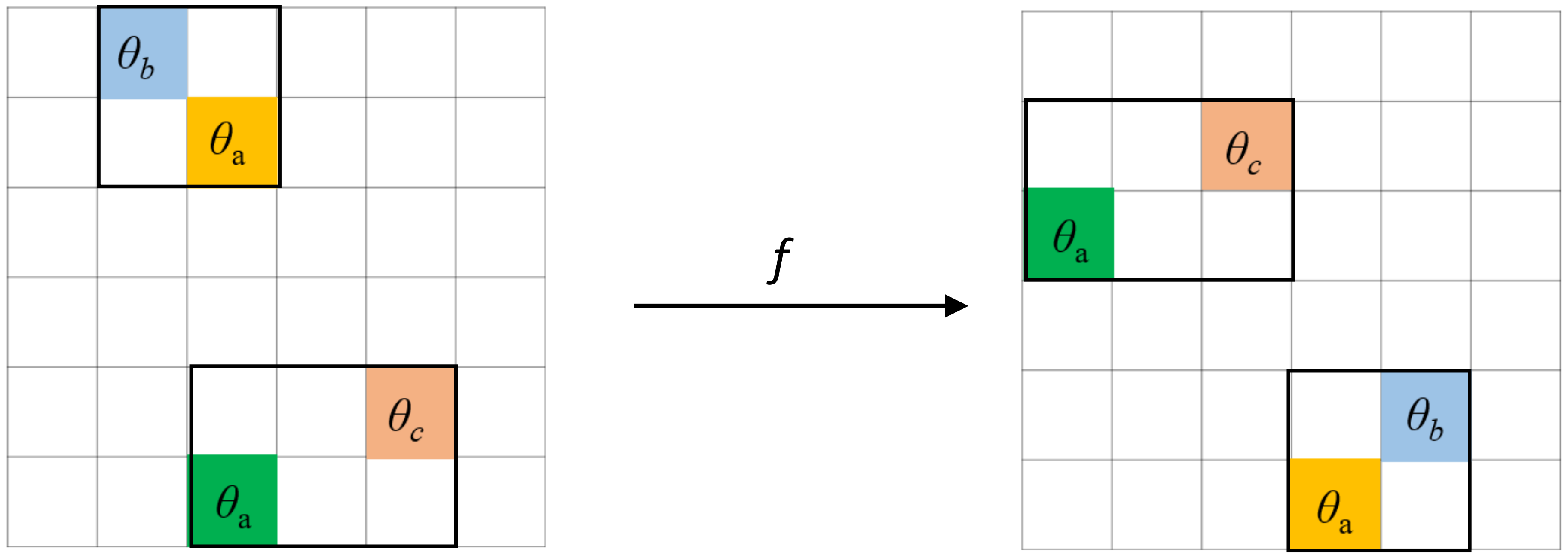
With mathematical thinking, a faster approach is possible…





Mathematical definition of **islands?**

An *island* is a group of molecules separated by at least *d* cells.
d is an integer (*d* = 3 in the picture)



Configuration a

Configuration b

Configurations *a* and *b* are different.

But if we

i. rotate the islands of *a*, and

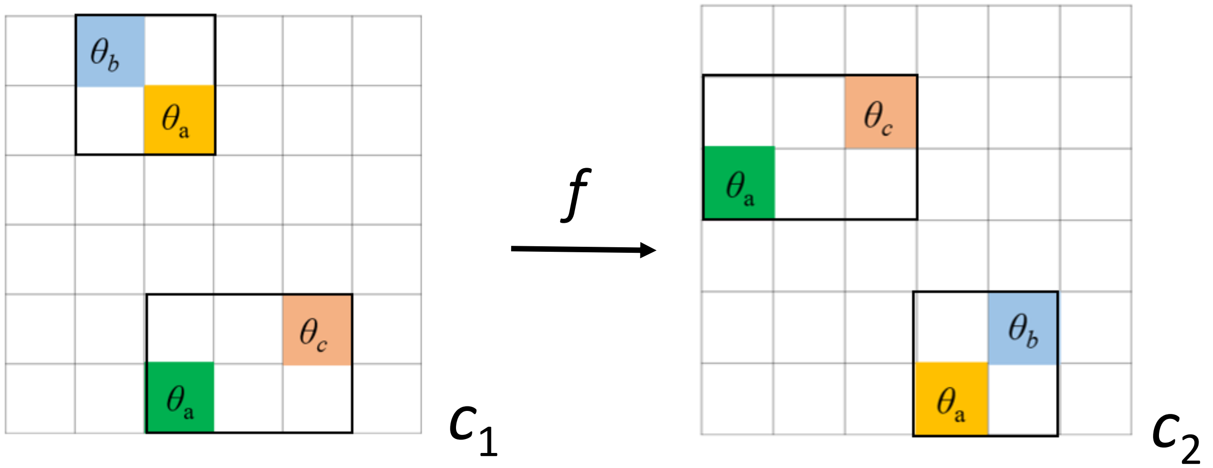
ii. move the islands of *a*,

f

we obtain configuration *b*. So configuration *a* and *b* contain the **same islands.**

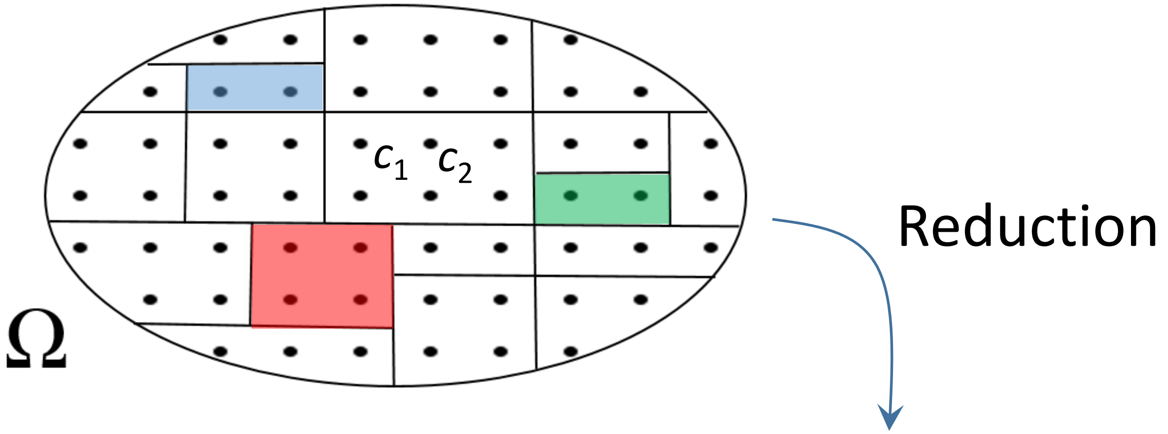
Space Reduction

The transformation *f* is called an **isomorphism.**



The isomorphism divides the configuration space into **equivalence classes**

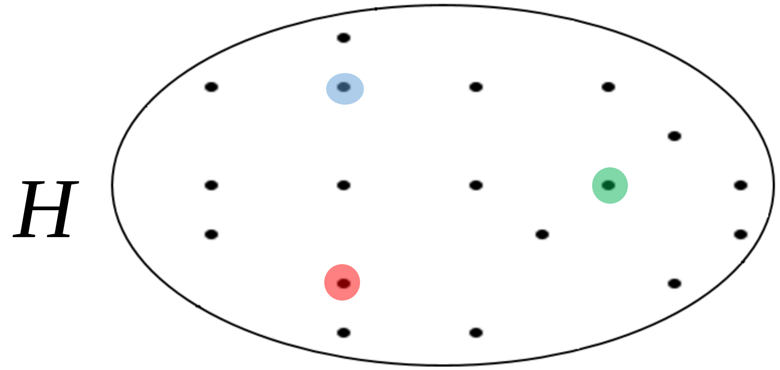
Reduced space H
= collection of equivalence classes



H has around 10^5 elements (10 molecule case)

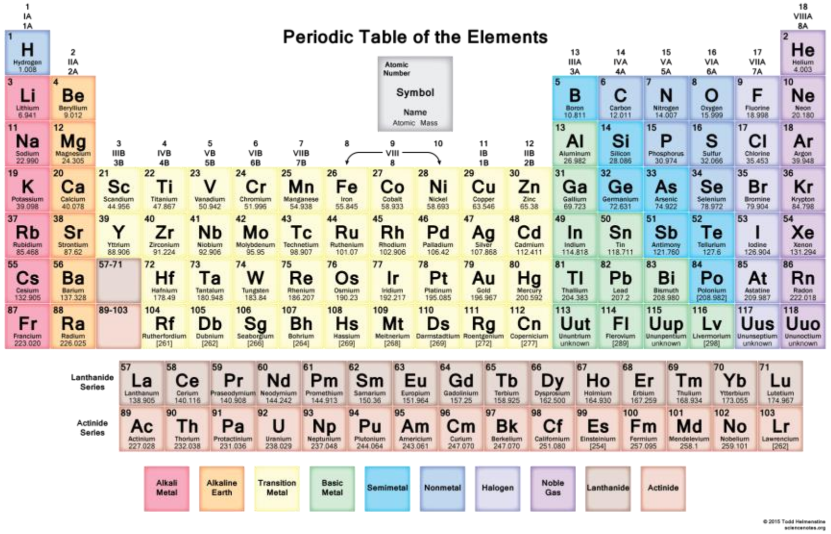
(compare: Ω has around 10^{23} elements!)

Time to check all probabilities ≈ 10^5 x 10^{-6} s ≈ 0.1s



Mathematical modeling for island formation

✓ 1. Identify the key parts of the real situation.



✓ 2. Abstract the real situation

✓ 3. Apply the laws of physics

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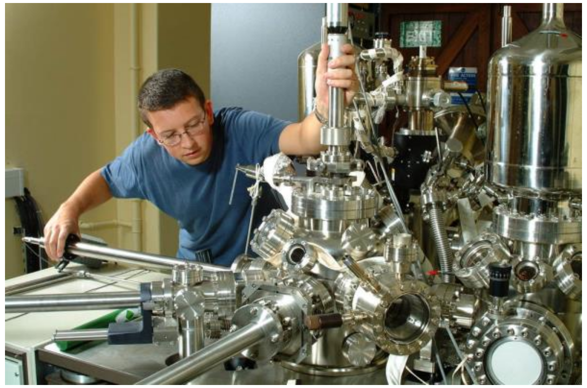
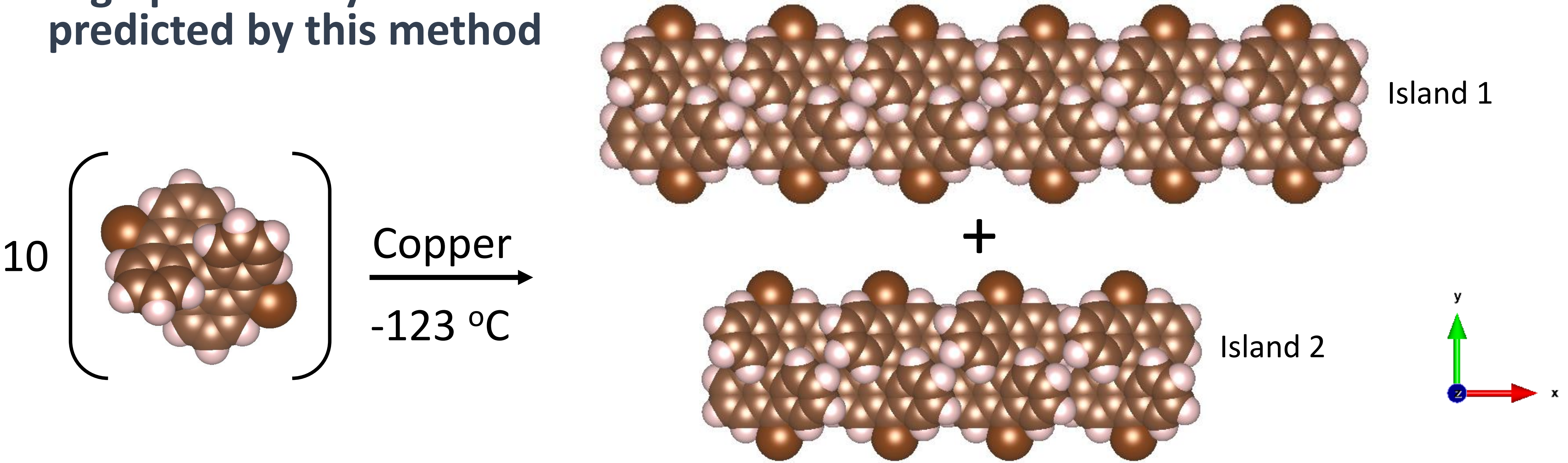
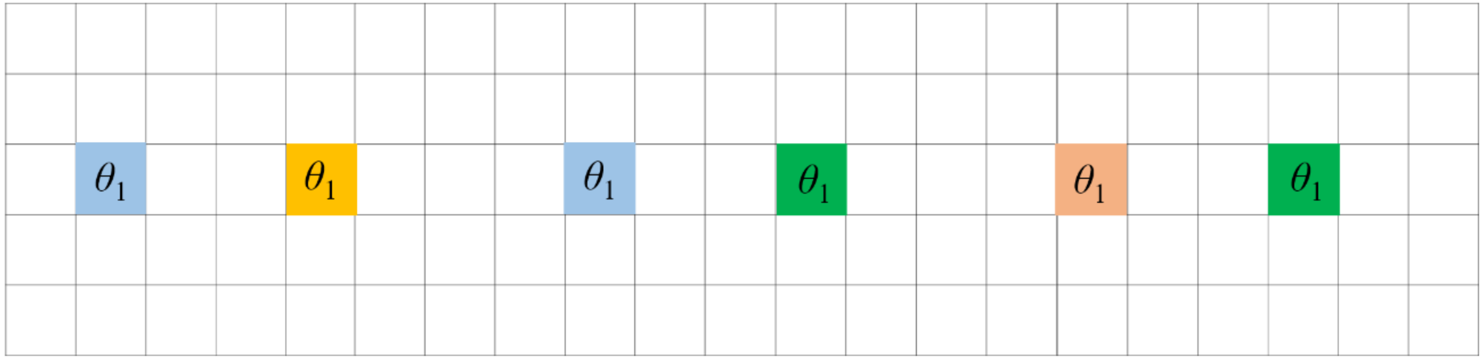
4. Compare to experiment



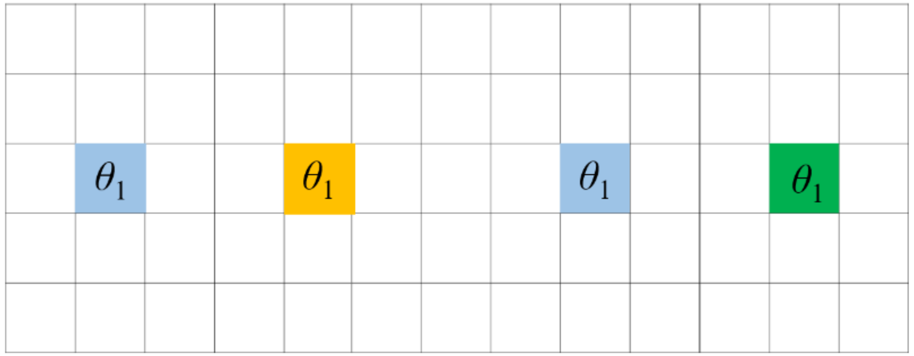
High probability islands predicted by this method



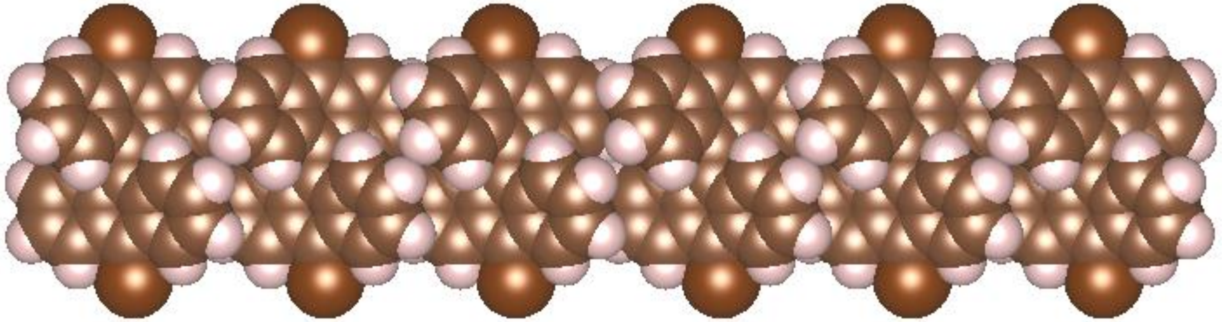
Island 1



Island 2

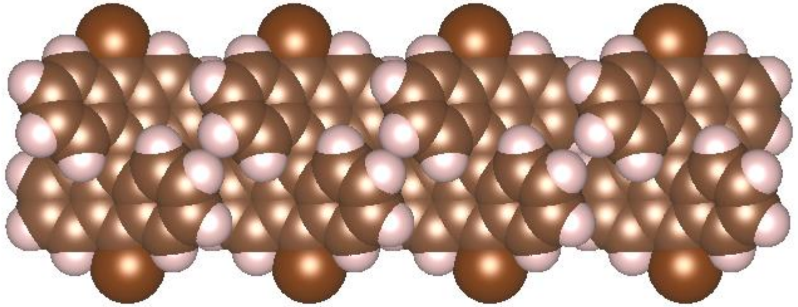


Mathematical modelling (-123°C)



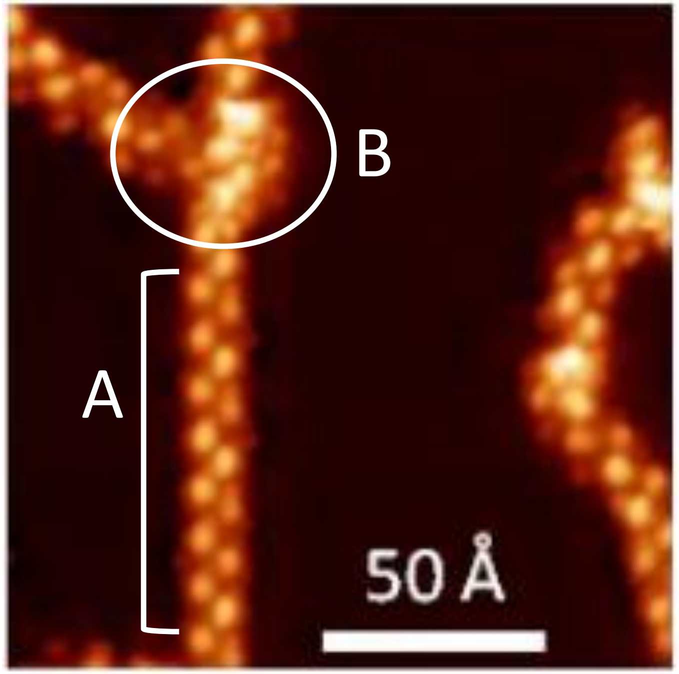
Island 1

+



Island 2

Experiment (30°C)

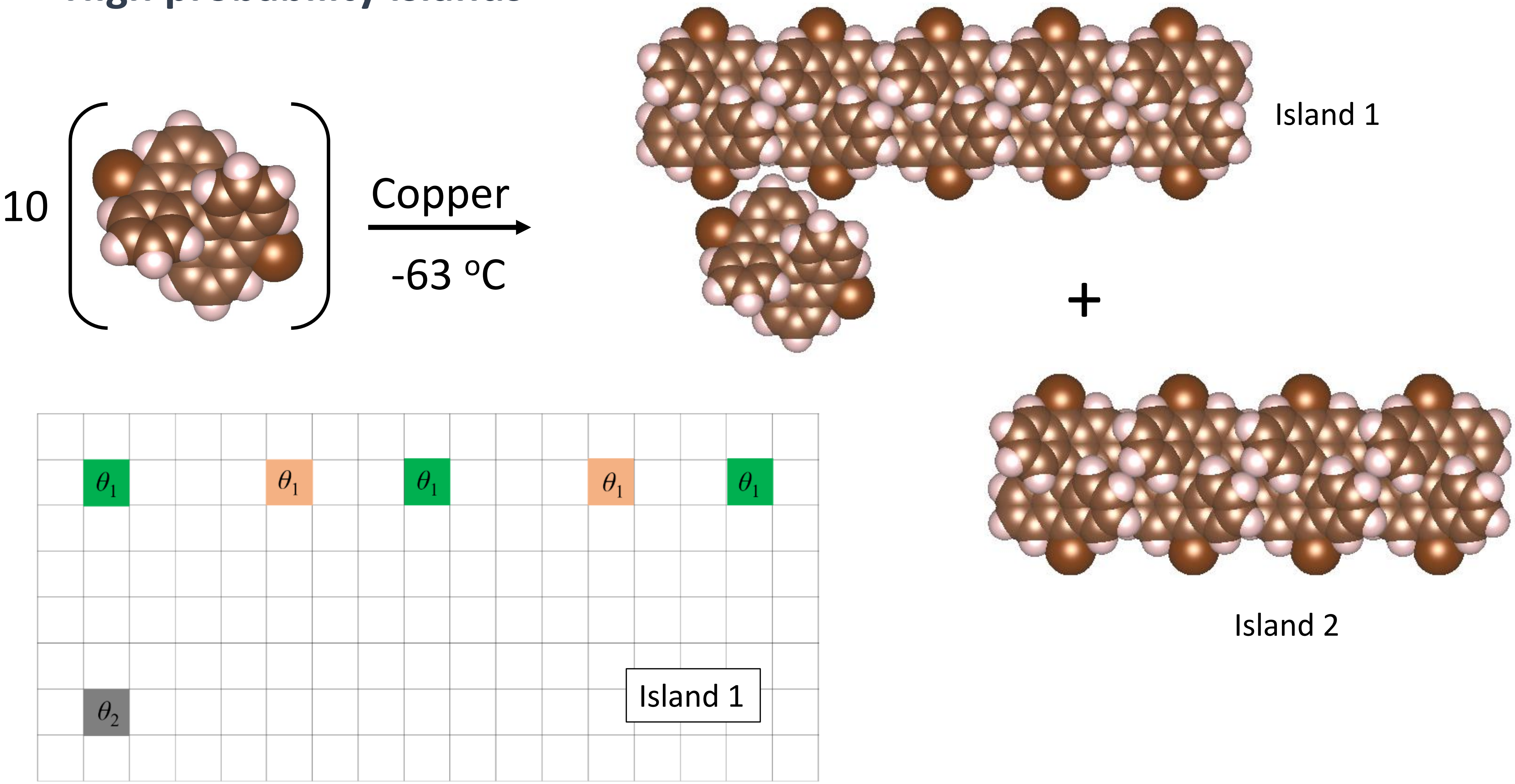


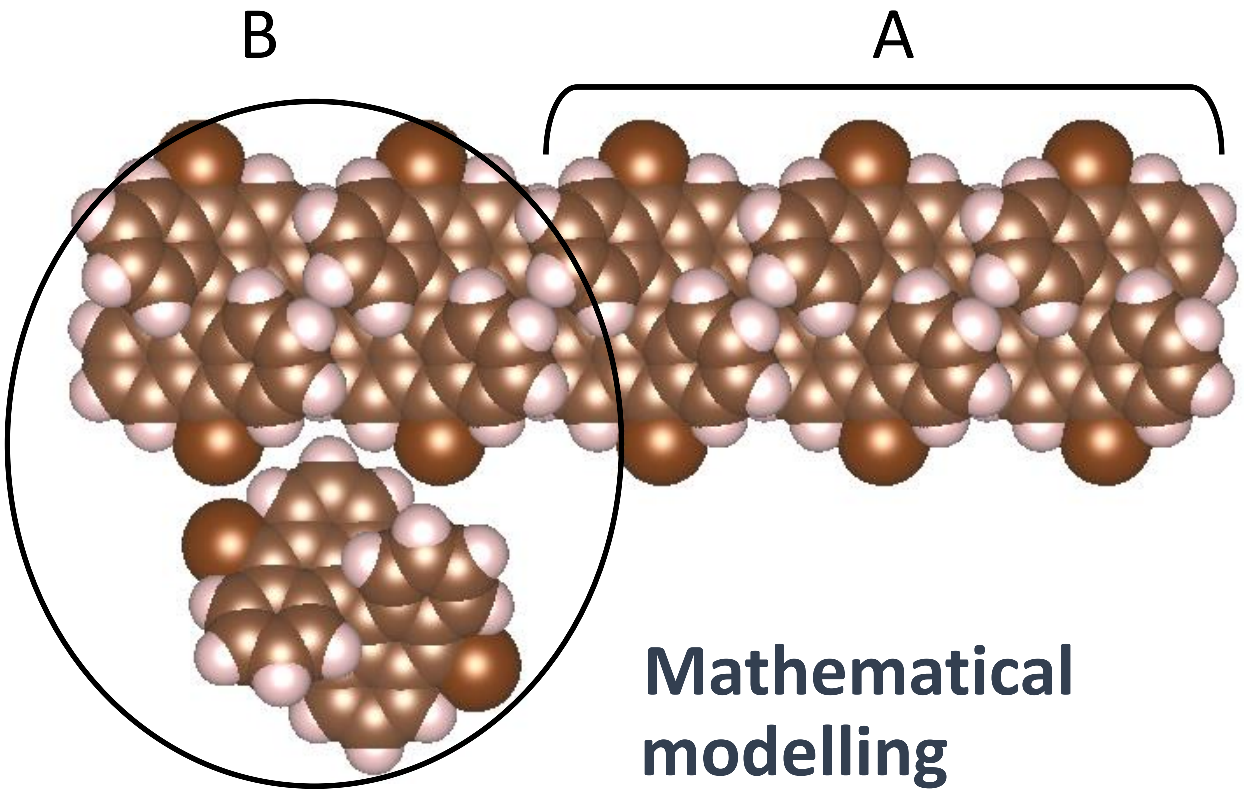
Good: Wire-shaped islands seen in experiment (A).

But: What about the part in (B)?

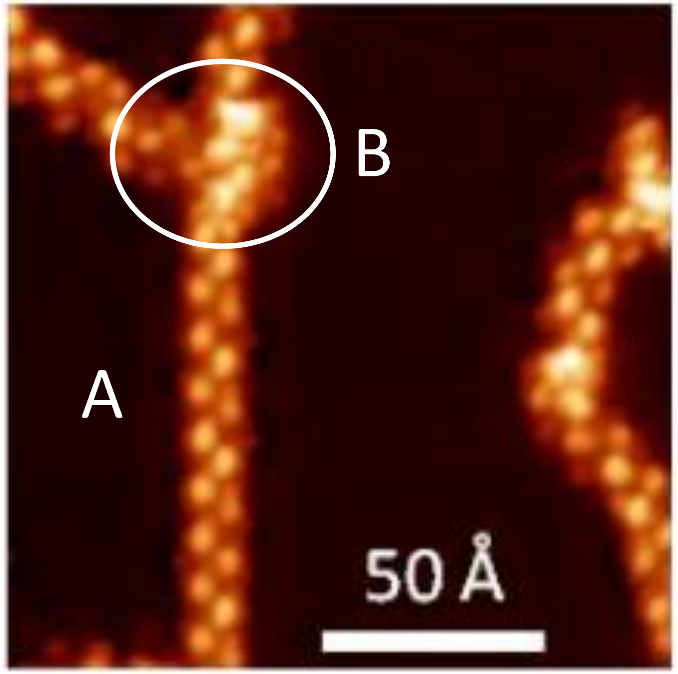
Let’s consider a higher temperature…

High probability islands





Mathematical modelling

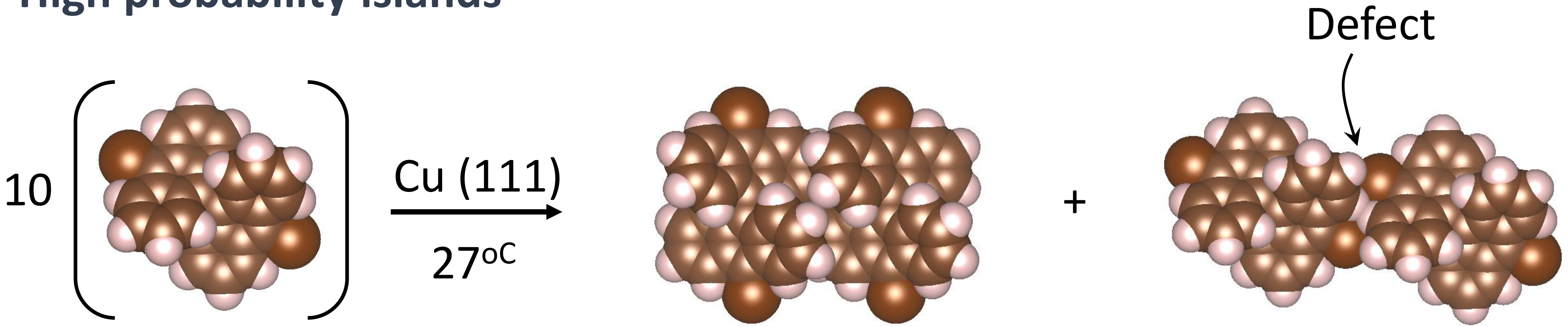


Experiment

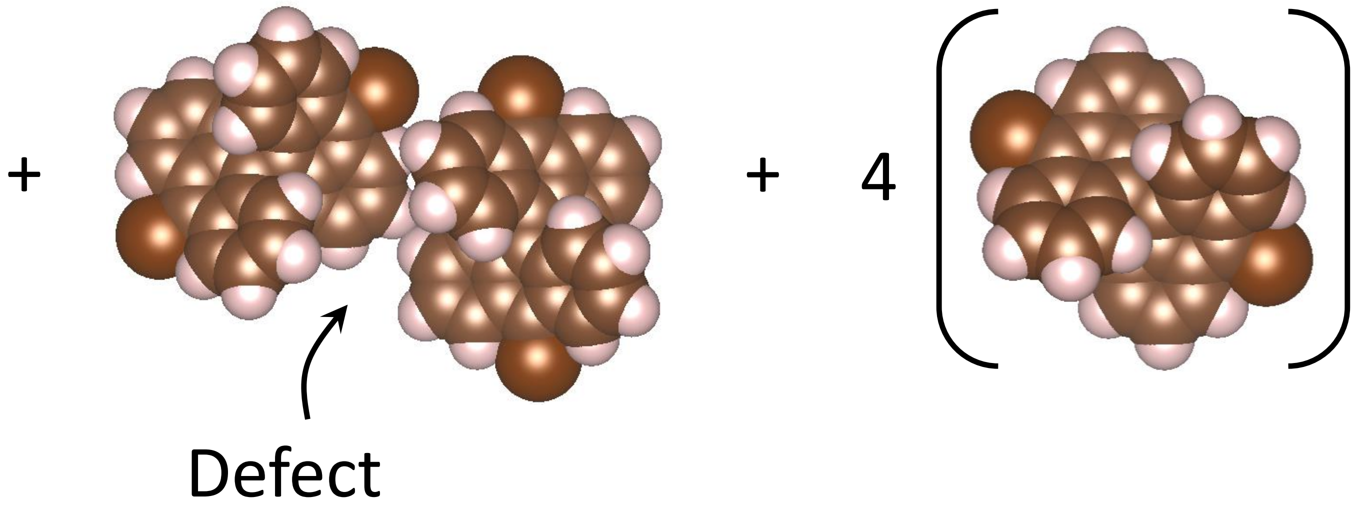
At higher temperatures, mathematical modeling predicts similar feature to (B).

What about even higher temperatures?

High probability islands



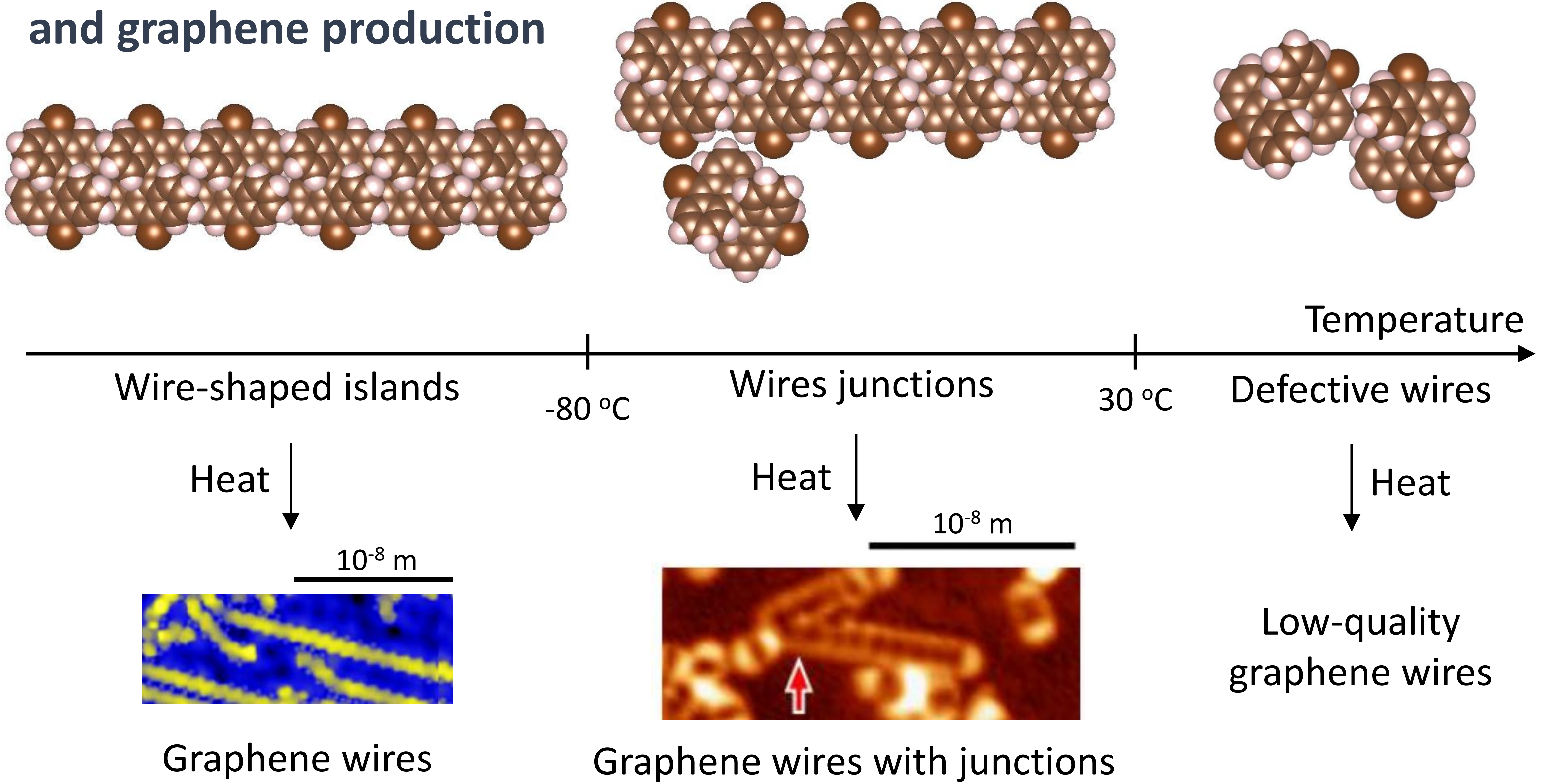
Island formation does not occur so much.



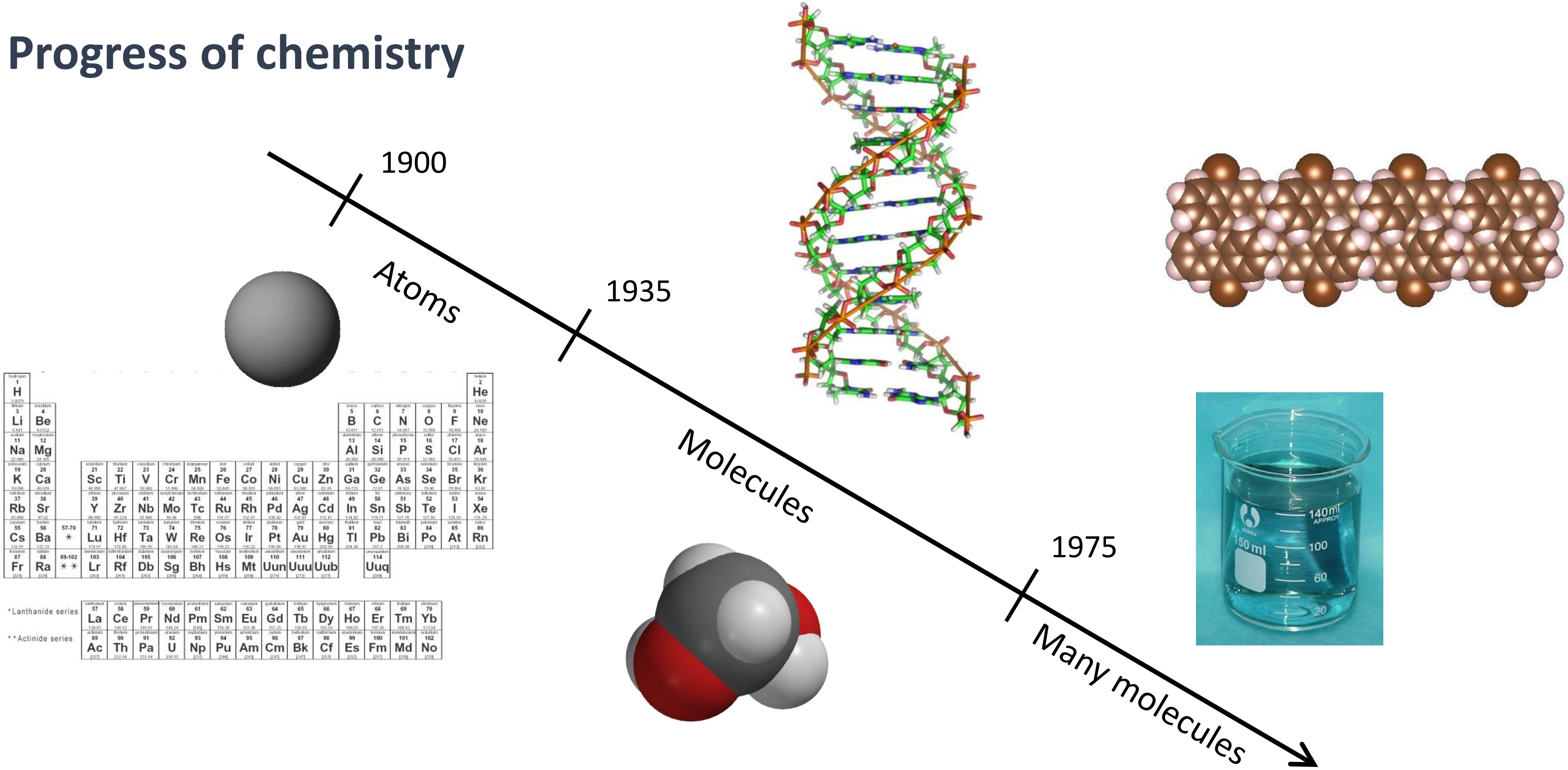
Islands are **short wires** with **defects**

Now, the rule starts to appear….

Rules for island formation and graphene production



Progress of chemistry



Rules concerning atoms and single molecules are **well understood**

Rules for interactions between molecules are **poorly understood**.

Mathematical modeling is necessary for progress in science.

Can you help? Study mathematics and science at high school and university.

Study them hard!

Study them critically and carefully!

Study them enthusiastically!

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